

Re: Kuratowski Ordered Pair

Source: <http://sci.tech-archive.net/Archive/sci.math/2007-12/msg02741.html>

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 - *Date:* Thu, 13 Dec 2007 04:53:37 -0800 (PST)
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Noel wrote:

Hero wrote:

Instead of asking directly what is meant by an ordered pair or n-tuple, I think it's interesting to ask instead what is involved in the so-called characteristic property. So we have some definition for an n-tuple, an ordered triple, say, (a, b, c), which gives us an expression using a, b, c, which we can write as $E[a, b, c]$, such that:

$$E[a, b, c] = E[u, v, w]$$

iff

$$a=u \text{ and } b=v \text{ and } c=w.$$

What does this mean? Shocking as this may seem, I do not think it has anything necessarily to do with order. What it guarantees is that we have a kind of differentiated triple, in the sense that in (a, b, c), should it turn out that $a=b$, then there are still two instances of a (or b). A definition satisfying the characteristic property has everything to do with ensuring that (a, b, c) cannot be equal to (a, b), for example, and nothing directly to do with order.

When we write:

$$(a, b, c) = (u, v, w)$$

only if we already understand that a comes first, b second, c third (likewise for u, v, w) do we know what we are equating with what. The characteristic property is not enough, or rather it already presupposes the essential order.

Get away from the use of consecutive letters.

$$(x, c, p) = (d, r, t)$$

If the characteristic property were enough, that $x=d$ and $c=r$ and $p=t$, how would we know not to write (x, c, p) as, say, (c, x, p), so that

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$$(x, c, p) = (d, r, t) = (c, x, p) = (p, c, x) = (t, d, r) \text{ etc}$$

and all the while we understand that $x=d$ and $c=r$ and $p=t$?

The characteristic property is not a defining characteristic of the (ordered) tuple, but a trivial and vacuous consequence employing the order.

Let's negate:

$$(a, b) = (p, q) \iff a = p \text{ OR } b = q$$

$a = p \text{ OR } b = q \implies \{ a, b \} = \{ p, q \}$ but it does not follow that a is before b

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There is possibly one other small consequence. The theory of sets might have difficulty in maintaining it has a foundation for arithmetic.

$(a, a) = (a)$, but how can it be that $a = a$ and that a is existing twofold?

Look at Aristotle for the most basic law of math.

And when $(a, b) = (b, a)$ compare this to the invariant of time.

With friendly greetings

Hero

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