Bessel Functions / Eigenvalues / Heat Equation

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Hello

Trying to calculate and simulate with Matlab the Steady State
Temperature in the circular cylinder I came to the book of Dennis G.
Zill Differential Equations with Boundary−Value Problems 4th edition
pages 521 and 522

The temperature in the cylinder is given in cylindrical coordinates
by:

u(r,z)= u_0 [Sum from n=1 to Infinite] of:

sinh(lambda_n*z) * J_0(lambda_n * r)

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lambda_n * sinh(4 * lambda_n) * J_1(2 *lambda_n}

My problems:
−I don't understand very well the Bessel Function either the
Eigenvalues and need a bit of help
−PDE Knowledge and simulations is basic

Information:
With the separation of variables method in cylindrical coordinates and
having U as temperature the equations are defined as follows:

Initial Conditions:
u(2,z)=0 0<z<4
u(r,0)=0 0<r<2

Boundary Condition:
u(r,4)=u_0 0<r<2

u=R(r)Z(z)

r*R" + R' + ((lambda)^2)*r*R = 0 Cauchy−Euler equation

Z" + 0 − ((lambda)^2) * Z = 0

With solutions:

R = c_1 * J_0 (lambda * r) + c_2 * Y_0 (lambda * r)
Z = c_3 * cosh(lambda * z) + c_4 * sinh(lambda * z)

The book states "the assumption that the function u is bounded at r = 0 demands that c_2 = 0"

The condition u(2,z) = 0 implies that R(2) = 0

The positive eingenvalues lambda_n of the problem are defined by:

J_0(2*lambda)=0

Now I come to my questions:
1.− What is meant by "the function u is bounded at r = 0"?

Is it right to understand that c_2 = 0 because the Bessel Function of the Second Kind of Order Zero (Y_0) tends to minus infinite while aproaching to r=0 from the right side, what is meant by bounded at r=0?

2.− I did some research on the Bessel Functions of the First and Second Kinds, solved the Bessel equation step by step and "more or less" understood it. My problem is that I don't understand neither how to calculate the eigenvalues lambda_n of the steady state temperature in a circular cylinder.

Does the equation J_0(2*lambda) = 0 means that:

2*lambda_{1} = 2.4048
2*lambda_{2} = 5.5201
2*lambda_{3} = 8.6537
..
..
..
lambda_{1} = 2.4048 / 2 ???

Or in words said: The eigenvalues are defined by the division by two of the x value where J_0 is a zero or a root?

3.− If we go back to the final solution there are two terms a J_0(lambda_n*r) and a J_1(2*lambda_n) and my goal is to implement this terms on Matlab to understand better the temperature U.

So my approach would continue trying to define in Matlab a vector for

J_0(lambda_n * r), is it right to think that having two vectors of the same size lambda_n and r, being r defined from 0 to 2, find out which is the value of the bessel function J_0 at say J_0((2.4048/2)*r)?

Unfortunately I can't write my post in a less complex way, hope it is understood, any help, hint or tip would be kindly appreciated.

Best Regards
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