Re: Kuratowski Ordered Pair

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- *From*: noel etters
- *Date*: Thu, 20 Dec 2007 21:34:47 +0000

On Tue, 18 Dec 2007 09:32:54 −0800 (PST), MoeBlee <jazzmobe@xxxxxxxxxxx>
wrote:

> On Dec 18, 2:34 am, noel etters wrote:
>
>> How on earth could you interpret the 'rigmarole' as an argument
>> about commutivity? You don't seem interested in trying to understand what
>> I
>> am saying.
>
> You don't seem intererested in what *I* am saying.
>
> I didn't say that you intend to argue by commutativity. However, I've
> pointed out that there is nothing substantive in your argument (or at
> least as you presented it) that can't be taken in by noting that
> ordered pairing is not commutative.

Then you haven't understood it at all.

>> The point is that the Kuratowski set does not provide a definition
>> of the ordered pair, as this is understood independently of set theory.
>
> There is no single (let alone rigorous) understanding of ordered pair
> independent of a mathematical theory.

Perhaps so. I am merely assuming that any reasonable understanding
of an ordered pair would involve some understanding of order.

> It's an informal notion and one
> that set theory captures in the ways that have been mentioned already.
>
> If your point is that there is some informal notion of which set

> theory doesn't capture every possible nuance, shade, connotation, or
> other aspect, then, of course, I don't know anyone who claims that a
> formal theory can capture all those aspects of an informal notion.
> That's a non−starter.

That's not my point.

> But the Kuratowski defintion does capture the exact DESIRED
> mathematical aspects.

No, it doesn't.

> I've explained that ad nauseam, and will do it
> again in this post since you keep skipping past that.

And I am being forced to explain ad nauseum why it doesn't.

> The Kuratowski definition is not INTENDED d to address any host of
> other informal notions. But even in informal terms, the Kuratowski
> definition is of an operation such that given an input of x and then
> y, the operation yields that x is first and that y is second. And the
> operation yields that <x y> = <z w> iff x=z and y=w, and that we can
> define "first in an ordered pair" and "second in an ordered pair", and
> that p is an ordered pair iff p = <1st(p) 2nd(p)>. Again, give me x,
> then give me y, and through the Kuratoswki defintion, I give you back
> that x is first and that y is second. That there is an "internal"
> peculairity in {{x} {x y}} just comes with the territory of
> mathematical definition in which one may recognize that, yes, the
> PARTICULAR construction may be arbitrary, since there may be more than
> one way to capture the"outer" structure we want. It's just finding a
> way of "coding" so that our ultimate concern is not in the particular
> way we coded but rather in the fact that the coding does capture what
> we INTEND to capture.

But it doesn't capture that.

The point of introducing the slotted pair, a pair in which there is no
first or second place or position, only distinct places, so that it is not
an ordered pair, though (a, b) not= (b, a), is to show where the
Kuratowski definition is appropriate. Again you need to be reminded that:

<x, y> = <z, w> iff x = z and y = w

cannot be constitutive of an ordered pair, since unless we are already assuming order, it is entirely consistent that (x, y) = (y, x) = (z, w) = (w, z) and x=z and y=w.

Again, I cannot see how to do better than repeat that since:

{{x}, {x, y}} = (x, y) Kura main, but
{{x}, {x, y}} = (y, x) Kura reverse

it is clear that this set cannot distinguish order, but by arbitrarily choosing the Kuratowski main or reverse interpretation we can choose an order, which is then in effect a conventional representation for a slotted pair. Another way to put this would be to say that we can represent a slotted pair in two equal and opposite, exclusive ways using ordered pairs. The slotted pair is indifferently either <a, b> or <b, a>, but not both. Distinctness but not order is important. This is exactly what the Kuratowski definition (and others like it) are good for.

> > Instead, it (possibly) provides a definition of a weaker notion, what I have called a slotted pair. In so far as SS' above (labels for the distinct
> > slots) represents an order (since it is a kind of lexical necessity that we
> > have either SS' or S'S, or that S and S' taken together be expressed with some particular orientation), this is the order carried or chosen by the (arbitrary) decision to go with the Kuratowski main set as opposed to its reverse

> Right there: "as opposed to the reverse". Yes, it is arbitrary, and that is what I mean by your argument reducing to ordered pairing not be commutative.

It is arbitrary in a particular and symmetric way, and in a way that has the consequences I have tried to set out.

> > −− it is not defined by the Kuratowski set, it is used by it. In fact all that is necessary for a slotted pair is that there are understood to be distinct slots.

> None of your rambling about slots and whatnot refutes that the Kuratowski definition does the job that it is INTENDED to do − capturing the needed mathematical aspects.

No it doesn't, because you haven't listened carefully enough to what I am saying about slotted pairs.

What mathematical result would you like to prove but can't prove due to some flaw in the Kuratowski definition? What is it mathematically that you feel you cannot express in the langauge of set theory?

The notion of an ordered pair or n−tuple without using number.

Again,
I confess I may have got carried away in other subthreads in suggesting tthere might be a huge flaw in set theory because of this. No doubt the slotted pair is entirely adequate to serve as a building block for the notions of relation, function and so on. But it is grossly misleading, at least, to suggest that the Kuratowski set provides a definition of the ordered pair.

In, say, Z set theories, any set theoretical definition other than the Kuratowski definition will ARRIVE at the same relevent theorems that I mentioned above.

We want a definition that yields those relevent theorems.

Since you like to think in metaphor:

Imagine that I give you x and you put some kind of mark on x and then I give you y and you put a different mark on y. And no matter what two inputs I give you, you mark them just as you marked x and y. Then you can always tell me which is first because the first has the mark for first and the second has the mark for second.

The Kuratowski definition acheives that by (metaphorically speaking here) putting {} around x as the marker for first and by putting the first (x) with {} around y to mark that y is second. Granted, it looks like a rather odd way of doing the marking, but it is efficient and it acheives exactly what we want of it.

The Kuratowski definition gives you the marks, not the interpretation 'first' and 'second'.

We could use this defintion instead:

<x y> = {{0 x} {1 y}}.

(Would you like that any better?)

Yet, both definitions yield the exact theorems that we INTEND. And what were interested in is working at the point from which we have those theorems. How we got our "coding" done to arrive at that point

> is not so important.
>
> If you want to axiomatize mathematics in some theory OTHER than set theory, then, yes, of course, the Kuratowski definition may not be relevent to your theory at all.
>
> > There is no wonder that others, such as Hero, want to talk about the actual properties of a real ordered pair,
>
> The "actual" properties? A "real" ordered pair? Where may I find this mysterious creature?

In ordinary mathematics.

> > and no wonder that the Kuratowski definiton generally tends to elicit some discomfort. But I suspect you have already dismissed any such discomforts with an airy wave.
>
> I've given detailed accounts. Too detailed. Those are not "airy waves". Though, I realize that, in your frustration that I don't happen to agree with you, you would like to condescendignly dismsiss my actual substantive accounts as "airy waves".

Detailed accounts? It comes across to me as a stormtrooping insistence on the orthodox which hasn't even heard the argument.

Noel
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