Re: Kuratowski Ordered Pair

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* *From*: MoeBlee <jazzmobe@xxxxxxxxxxx>
* *Date*: Thu, 20 Dec 2007 15:19:50 −0800 (PST)

On Dec 20, 1:34 pm, noel etters wrote:

> On Tue, 18 Dec 2007 09:32:54 −0800 (PST), MoeBlee <jazzm...@xxxxxxxxxxx>
> wrote:
>
>> I didn't say that you intend to argue by commutativity. However, I've
>> pointed out that there is nothing substantive in your argument (or at
>> least as you presented it) that can't be taken in by noting that
>> ordered pairing is not commutative.
>
> Then you haven't understood it at all.

I don't think so, but it is possible.

>>> The point is that the Kuratowski set does not provide a
>>> definition
>>> of the ordered pair, as this is understood independently of set
>>> theory.
>>
>> There is no single (let alone rigorous) understanding of ordered pair
>> independent of a mathematical theory.
>
> Perhaps so.

And if indeed not so, then there's little sense to your "as [oredered
pair] is understood independently of set theory".

> I am merely assuming that any reasonable understanding
> of an ordered pair would involve some understanding of order.

Sure, and it is from such an understanding that the definition was devised adopted and is understood by students since then.

> > It's an informal notion and one that set theory captures in the ways that have been mentioned already.
>
> > If your point is that there is some informal notion of which set theory doesn't capture every possible nuance, shade, connotation, or other aspect, then, of course, I don't know anyone who claims that a formal theory can capture all those aspects of an informal notion. That's a non−starter.
>
> That's not my point.

I take it that your point then is that there is some aspect of the notion of ordered pair that the Kuratowski definition does not capture. (If that is not a correct understanding of your point, then you'll have to correct me.)

But so far you've not shown a single mathematical concern that is not expressible by the set theoretical definitions. You talk about various metaphorical things, but still, I'm waiting to hear from you as to what specific mathematical result you feel is not expressible in set theory. (I see now that later in your post you do get around to addressing this.)

> > But the Kuratowski defintion does capture the exact DESIRED mathematical aspects.
>
> No, it doesn't.

Look, if you're just going to keep saying essentially, "No you can't, yes I can", then there's no discussion I can even address.

The exact desired mathematical aspects are to prove the basic theorems of ordered pairs, to define 'relation', 'function', 'Cartesian product', 'n−tuple', 'sequence', and then to use those for general matheamtics. And that is accomplished. So you're just saying "No" is not a refutation of that.

> > I've explained that ad nauseam, and will do it
> > again in this post since you keep skipping past that.
>
> And I am being forced to explain ad nauseum why it doesn't.

Where EXACTLY have you explained that the above mentioned purposes are not acheived?

> > The Kuratowski definition is not INTENDED d to address any host of other informal notions. But even in informal terms, the Kuratowski definition is of an operation such that given an input of x and then y, the operation yields that x is first and that y is second. And the operation yields that <x y> = <z w> iff x=z and y=w, and that we can define "first in an ordered pair" and "second in an ordered pair", and that p is an ordered pair iff p = <1st(p) 2nd(p)>. Again, give me x, then give me y, and through the Kuratoswki defintion, I give you back that x is first and that y is second. That there is an "internal" peculairity in {{x} {x y}} just comes with the territory of mathematical definition in which one may recognize that, yes, the PARTICULAR construction may be arbitrary, since there may be more than one way to capture the"outer" structure we want. It's just finding a way of "coding" so that our ultimate concern is not in the particular way we coded but rather in the fact that the coding does capture what we INTEND to capture.
>
> But it doesn't capture that.

You should just post one ASCII art illustration of you holding your hands over your ears and captioned, "No, no, no. I'm right. You're wrong."

> The point of introducing the slotted pair, a pair in which there is no first or second place or position, only distinct places,

Metaphorical. If you want a theory for slotted pairs, then go ahead and make one. (And I'll address your remark near the end of your post about what mathematics you feel is not acheived.)

> so that it is not
> an ordered pair, though (a, b) not= (b, a), is to show where the
> Kuratowski definition is appropriate. Again you need to be reminded that:
>
> <x, y> = <z, w> iff x = z and y = w

> cannot be constitutive of an ordered pair, since unless we are already assuming order, it is entirely consistent that (x, y) = (y, x) = (z, w) = (w, z) and x=z and y=w.

So what if that is consistent? What MATHEMATICS is not expressible on account of:

If x=z=y=w then

<x y> = <y x> = <z w> = <w z>.

That follows merely from plain identity theory.

> Again, I cannot see how to do better than repeat that since:
>
> {{x}, {x, y}} = (x, y) Kura main, but
> {{x}, {x, y}} = (y, x) Kura reverse
>
> it is clear that this set cannot distinguish order,

It's just a non sequitur that order is not captured.

One is the Kuratowski definition and the other can be expressed through the notion of converse.

And you didn't answer my analogy:

x^y = x mulipilied y times. Call that expo main.

x#y = y multiplied x times. Call that expo reverse (I chose '#' as the symbol).

So an operation can be defined in either "direction". That doesn't make the first defintion lacking!

> but by arbitrarily
> choosing the Kuratowski main or reverse interpretation we can choose an order, which is then in effect a conventional representation for a slotted pair.

Okay, so let "slotted pair" be defined as either Kuratowski or Kuratowski−main. One or the other, not both.

> Another way to put this would be to say that we can represent a slotted pair in two equal and opposite, exclusive ways using ordered pairs.

Another way to put this would be to say we can represent slotted expo
in two opposite, exclusive ways using multiplication iterated.

> The slotted pair is indifferently either <a, b> or <b, a>, but not both.

If it's a proper definition than it is always one definition. Take
your pick.

> Distinctness but not order is important. This is exactly what the Kuratowski definition (and others like it) are good for.

You say this is what Kuratowski is "GOOD" for.

(And I'll address your remark near the end of your post about what
mathematics you feel is not acheived.)

> > > Instead, it (possibly) provides a definition of a weaker
> > > notion, what I
> > > have called a slotted pair. In so far as SS' above (labels for
> > > the
> > > distinct
> > > slots) represents an order (since it is a kind of lexical
> > > necessity that
> > > we
> > > have either SS' or S'S, or that S and S' taken together be
> > > expressed with
> > > some particular orientation), this is the order carried or
> > > chosen by the
> > > (arbitrary) decision to go with the Kuratowski main set as
> > > opposed to its
> > > reverse

> > Right there: "as opposed to the reverse". Yes, it is arbitrary, and that is what I mean by your argument reducing to ordered pairing not be commutative.

> It is arbitrary in a particular and symmetric way, and in a way that has the consequences I have tried to set out.

So the Kuratowski definition has consequences (in the sense that any defintion has eliminable consequences, in the sense of the eliminability of defined terms).

The Kuratowski definition, since it is a proper definition, introduces no contradiction in the theory. The Kuratowski definition allows for the abbreviation of notation and the choice of the particular definiens allows for mathematics to express what we're after − orderings, linear orderings, Cartesian planes, etc. So is there something "wrong" in any of those consequences? If so, what exactly is "wrong"?

> > > −− it is not defined by the Kuratowski set, it is used by it. In fact all that is necessary for a slotted pair is that there are understood
> > > to be distinct slots.

> > None of your rambling about slots and whatnot refutes that the Kuratowski definition does the job that it is INTENDED to do − capturing the needed mathematical aspects.

> No it doesn't, because you haven't listened carefully enough to what I am saying about slotted pairs.

I've listened carefully. No matter how hard I'd listened, you've not shown what the Kuratowski definition does not do that it was INTENDED to do.

Please, ante up already. Please say EXACTLY what the Kuratowski definition was INTENDED to do but does not do.

> > What mathematical result would you like to prove but can't prove due to some flaw in the Kuratowski definition? What is it mathematically that you feel you cannot express in the langauge of set theory?

> The notion of an ordered pair or n−tuple without using number.

Finally! Okay.

(1) The Kuratowski definition of ordered pair does not use number. So that's done.

(2) How could n−tuple not use number where 'n' is a variable
understood in context to range over natural numbers?!

(3) As to n−tuples, for any GIVEN n, we can define the n−tuple
operation without numbers. But for defining 'n−tuple' in GENERAL, it
would be pretty awkard at least without numbers. And so for YOUR
proposal too. If you are to GENERALIZE over n, then you've not shown
how to do that without generalizing over n as a numbers.

(4) Why is is important to define without reference to numbers?

(5) Numbers are definable in set theory anyway, so wherever we do
refer to numbers, we could instead use even just the PRIMITIVE of set
theory. Wherever we use numbers, we could instead plug in the
definiens for that number or, as the case may be, for in general
'natural number'. In that sense, we could collapse ALL of our
definitions back to the primitive.

(6) You've still not shown that the Kuratowski definition fails what
it was inteneded for.

> Again,
> I confess I may have got carried away in other subthreads in
> suggesting tthere might be a huge flaw in set theory because
> of this. No
> doubt the slotted pair is entirely adequate to serve as a
> building block
> for the notions of relation, function and so on. But it is
> grossly
> misleading, at least, to suggest that the Kuratowski set
> provides a
> definition of the ordered pair.

In, say, Z set theories, any set theoretical definition other than the
Kuratowski definition will ARRIVE at the same relevent theorems that I
mentioned above.

We want a definition that yields those relevent theorems.

Since you like to think in metaphor:

> > Imagine that I give you x and you put some kind of mark on x and then I give you y and you put a different mark on y. And no matter what two inputs I give you, you mark them just as you marked x and y. Then you can always tell me which is first because the first has the mark for first and the second has the mark for second.
>
> > The Kuratowski definition acheives that by (metaphorically speaking here) putting {} around x as the marker for first and by putting the first (x) with {} around y to mark that y is second. Granted, it looks like a rather odd way of doing the marking, but it is efficient and it acheives exactly what we want of it.
>
> The Kuratowski definition gives you the marks, not the interpretation 'first' and 'second'.

I don't know what you mean by a definition giving a 'mark'. A definition defines a symbol, yes. That's the nature of formal definition. Interpretation is another matter. I don't know in what sense you think there is some other way of establishing an interpretation with regard to formal definitions.

> > We could use this defintion instead:
>
> > <x y> = {{0 x} {1 y}}.
>
> > (Would you like that any better?)
>
> > Yet, both definitions yield the exact theorems that we INTEND. And what were interested in is working at the point from which we have those theorems. How we got our "coding" done to arrive at that point is not so important.
>
> > If you want to axiomatize mathematics in some theory OTHER than set theory, then, yes, of course, the Kuratowski definition may not be relevent to your theory at all.

> > > There is no wonder that others, such as
> > > Hero, want to talk about the actual properties of a real
> > > ordered pair,

> > The "actual" properties? A "real" ordered pair? Where may I find this
> > mysterious creature?

> In ordinary mathematics.

Ordinary mathematics is formally axiomatized by set theory. As to
ordinary mathematics in its native informal state, I know of no
specification of ordered pair other than as primitively understood or
as defined in informal set theory. (You may inform me otherwise.
Please point me to a text in ordinary mathematics that describes
properties of ordered pairs inconsistent with the Kuratowski
definition.) Meanwhile, you've still not shown anything done with
ordered pairs in ordinary mathematics that can't be done with the
Kuratowski definition.

> > > and
> > > no wonder that the Kuratowski definiton generally tends to
> > > elicit some
> > > discomfort. But I suspect you have already dismissed any
> > > such discomforts
> > > with an airy wave.

> > I've given detailed accounts. Too detailed. Those are not "airy
> > waves". Though, I realize that, in your frustration that I don't
> > happen to agree with you, you would like to condescendignly dismsiss
> > my actual substantive accounts as "airy waves".

> Detailed accounts? It comes across to me as a stormtrooping
> insistence on the orthodox which hasn't even heard the argument.

(1) Again, you CHARACTERIZE, now as "stromtrooping (yeah, right, in a
disagreement on mathematics, it's always a good idea to start
characterizing the other guy as NAZI−like) insistence on the orthodox"
without saying what is the least bit incorrect in the accounts.

(2) I don't insist on the orthodox in any sense except to point out
that you've not shown any particular mathematical sense of order that
is not handled. I am happy to anyone propose an alternative system.

But you haven't provided one.

(3) That I don't find your arguments convincing does not entail that I haven't heard them or that I haven't tried to understand them.

MoeBlee

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