Re: Real numbers , what are they good for ?

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- *From*: mjc <mjcohen@xxxxxxx>
- *Date*: Fri, 21 Dec 2007 14:11:17 −0800 (PST)

On Dec 21, 1:06 am, Ray Vickson <RGVick...@xxxxxxx> wrote:

> On Dec 20, 4:27 pm, "FredJeffr...@xxxxxxxxx" <FredJeffr...@xxxxxxxxx>
> wrote:
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> > On Dec 16, 2:37 pm, jane <jane1...@xxxxxxxxxx> wrote:
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> > > A bit shaky question :
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> > > A friend of mine was asked the following question:
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> > > What are real numbers good for ?
> >
> > > How would you answer this question, stressing properties
> > > which, differ exactly real numbers and which are useful in
> > > analysis ?
> >
> > > I can give my own answer as well, but i would like to see
> > > other opinions, perhaps written shorter and having more
> > > information.
> >
> > > Thanks.

> I came across the following which may be of interest: From Alexandre Borovik's blog http://dialinf.wordpress.com/2007/12/17/commented−out/ referring to Vladimir Arnold's "What is Mathematics"
>
> Vladimir Arnold forcefully stated in one of his books that it is wrong to think about finite difference equations as approximations of differential equations. It is the differential equation which approximates finite difference laws of physics;

As far as we know, the laws of physics are described /exactly/ by differential equations, and so finite−difference approaches are approximations. Arnold seems to have it wrong, if he said what you claim. I have not read his blog.

> it is the result of
> taking an asymptotic limit at zero. Being an approximation, it is easier to solve and study.

It is not an approximation, as far as we know.

> In support to his thesis, Arnold refers to a scene almost everyone has seen: old tires hanging on sea piers to protect boats from bumps. If you control a boat by measuring its speed and distance from the pier and select the acceleration of the boat as a continuous function of the speed and distance, you can come to the complete stop precisely at the wall of the pier, but only after infinite time: this is an immediate consequence of the uniqueness theorem for solutions of differential equations. To complete the task in sensible time, you have to allow your boat to gently bump into the pier. The asymptotic at zero is not always an ideal solution in the real world. But it is easier to analyze!

OK, so in a situation of human (or machine) control we are limited in accuracy, and may need to be satisfied with a discrete approximation of the true continuous−time, continuous−space situation. However, what about those cases where human or machine control does not enter, such as the orbits of planets, the behaviour of electrons within atoms, etc.? Are you (or Arnold) claiming that "Nature" is discrete?

R.G. Vickosn

Yes, but look at how these differential equations are derived:

Equations are derived for finite step sizes, and the step sizes are allowed to approach zero. In other words, the differential equations are limits of difference equations. So the difference equations come first!
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