Re: Is it possible to generalise the Galois group definition for certain sets of polynomials

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- *From*: Chip Eastham <hardmath@xxxxxxxxx>
- *Date*: Fri, 28 Dec 2007 07:38:18 −0800 (PST)

On Dec 28, 10:06 am, Gerry <Gerry...@xxxxxxxxx> wrote:

> Hi all,
>
> is it possible to generalise the Galois group definition for the
> following sets of polynomials
>
> P(x)=x^(6+4*n)+c*x^(2+4*n)−1
>
> and
>
> Q(x)=P(x)+2
>
> n >= 0
> c any complex number ;−)
>
> with roots pi = ai + bi*I for polynomial P(x)
> and roots qi = bi + ai*I for polynomial Q(x)
>
> i=1 .. 4*n+6 the index for the roots
> I=sqrt(−1)
>
> Earlier this year there was a post on this subject under the
> description:
> "Transformation of complex arguments in polynomials" (June 22 Jürgen
> Will)
> I'm not sure how far they got but coincidentally i noticed some
> polynomials
> with this property and hence this post.
>
> As an example:
>
> For n=0, c=5 we get
> P(x)=x^6+5*x^2−1
> Q(x)=x^6+5*x^2+1
>
> For n=2,c=1/3 we get
> P(x)=x^14+1/3*x^10−1

> Q(x)=x^14+1/3*x^10+1
>
> Any comments are welcome.
>
> Gerry

I'm not sure what in the way of generalization you are looking for. The Galois group depends on an extension of fields. Often the base field includes coefficients of a polynomial while the extension field includes all the roots of the polynomial in a minimal fashion over the base field, and the Galois group of the polynomial is then the group of field automorphisms of the extension field which leave the base field fixed.

One possible quirk suggested by your post is that you include a coefficient c which can be complex. So it might be that you are thinking of the base field being something other than the rationals Q.

But your examples do not bear out that suggestion, as you take rational or integer values for c.

BTW, as you have set them up, there is a simple relationship between roots of P(x) and Q(x) in that Q(x) = −P(ix) when c is a real number.

P(x)=x^(6+4*n)+c*x^(2+4*n)−1

P(ix)=−x^(6+4*n)−c*x^(2+4*n)−1

−P(ix) = P(x)+2 = Q(x)

regards, chip

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