

Re: Limits

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- *From:* "Dave L. Renfro" <renfr1dl@xxxxxxxxxx>
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Dave L. Renfro wrote (in part):

Less well known are the 5 logarithmic indeterminate forms: $(\log_0)(0)$, $(\log_1)(1)$, $(\log_0)(\infty)$, $(\log_\infty)(0)$, and $(\log_\infty)(\infty)$.

David W. Cantrell wrote (in part):

There are 5 because there would have been 5 indeterminate forms involving division in your first list if signs had been shown. The logarithmic forms correspond with

$-\infty/-\infty$, $0/0$, $\infty/-\infty$, $-\infty/\infty$ and ∞/∞

Thanks for your comments and, if I ever put any time into investigating this, I'll keep them in mind. A number of years ago (not really that long, but it was probably between 6 and 8 years ago) I came across a comment about these logarithmic forms. I didn't jot down the reference because I didn't expect it to be so rarely mentioned (I've never seen any mention of this idea since then, in fact), but I did write the idea down on a post-it-note that I put into a folder I have in which I toss in interesting indeterminate limit examples when I come across them (one of over a hundred such folders of topics that have relevance to things that often came up in my teaching or other things that I happen to be interested in) and I came across it yesterday when I pulled the folder out to get some examples. I'd pretty much forgotten about it (the logarithmic forms) until I saw it, but I'm still pretty sure I haven't come across the idea since I wrote the note about them.

Anyway, the main reason I'm responding is about something else that you and others (Ioannis Galikdas, Robert Israel, etc.) might be interested in. I recently came across the following

book:

Isaac Joachim Schwatt, "An Introduction to the Operations with Series", The Press of the University of Pennsylvania, 1924, x + 287 pages.

A second edition was published by Chelsea Publishing Company in 1962 (which I think was just a reprint with corrections), and reviews of it that I know of are: Amer. Math. Monthly 32 (1925), p. 383; Mathematics of Computation 17 (1963), pp. 91–92; Amer. Math. Monthly 70 (1963).

This book is a virtual gold mine of all sorts of exotic formulas. The first chapter (about 30 pages) consists of a large number of expressions for the n 'th derivatives of various functions, and the later chapters become more involved. Rather than try to describe the kinds of things in this book, look at the various conference abstracts of Schwatt's in the Bulletin of the American Mathematical Society:

<http://tinyurl.com/242q4n>

Dave L. Renfro

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