Re: Long−term behaviour of ODEs

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- *From*: Robert Israel <israel@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>
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Richard Hayden <r.hayden@xxxxxxxxx> writes:

> Hi,
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> If I have an autonomous locally Lipschitz system of n coupled (non−linear) ODEs, for which all (unique) solutions are bounded to remain in some compact subset of \real^n (and thus all solutions exist for all t >= t_0), am I guaranteed that as t −> \infty, solutions will converge to some equilibrium solution? If so, where can I find this result? I have a feeling I read such a result (or something similar) a while back but can't remember where.
>
> If not, is there usual way to prove that all solutions to an ODE system do exhibit this behaviour? That they always do is clear numerically but I've no idea how to go about proving it (if the above result does not hold). An example might be:
>
> da/dt = −a / (a + b) + c
> db/dt = −b / (a + b) + a / (a + b)
> dc/dt = −c + b / (a + b)
>
> with initial conditions a_0, b_0, c_0 > 0.
>
> The class of ODEs in question model a fixed population, i.e. satisfy x'_1(t) + ... + x'_n(t) = 0 for all times t >= t_0 in case this helps. Similar ODEs I presume arise in biological/chemical context, but I can't find any useful results.

Since you have the conservation law a + b + c = constant, this reduces your 3 x 3 system to a 2 x 2 system. The Poincare−Bendixson theorem then applies: the only possible limiting behaviours of a bounded trajectory involve approaching a fixed point, a limit cycle or a homoclinic orbit. And for larger n you also have the possibility of strange attractors and chaotic solutions.

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