Re: Dense vs. Continuous

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- *From*: Dave Seaman <dseaman@xxxxxxxxxxxx>
- *Date*: Tue, 15 Jan 2008 03:39:04 +0000 (UTC)

On Mon, 14 Jan 2008 18:20:09 −0600, The poster formerly known as Colleyville Alan wrote:

> "Dave L. Renfro" <renfr1dl@xxxxxxxxx> wrote in message
> news:8ad65a97−db49−465d−a88b−2be9a244e1e8@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
>
>> Dave L. Renfro wrote (in part):
>>
>>> I have the book, and I'm somewhat
>>> familiar with it, but it's at home and I'm not.
>>
>> Oh, I fogot. It's freely available in digital form now.
>> So let me rephrase this as "I'm not sufficiently motivated
>> in resolving the issue of whether the authors used "continuous"
>> in the context of describing sets to pursue the matter".
>>
>> Dave L. Renfro
>
> To clarify, the authors never mentioned the term "continuous", they
> mentioned that the rationals were "dense". I was trying to ascertain what
> the difference was between "dense" and "continuous". I was (am)uncertain as
> to whether even trying to compare "dense" and "continuous" is a valid
> question or if it is an apples−to−oranges type of question.

We can't hope to determine the difference between two terms unless the
terms in question have previously been defined. We have a definition for
a dense set, but there is no commonly accepted definition for a
continuous set. As previously noted, it makes sense to speak of
functions being continuous, but not *sets*.

> So, let me try again. If I have a function $y = f(x) = x$, the graph would be a 45* line through the origin in a standard Cartesian grid. The graph would be a continuous line with no breaks since it would be using the reals as the domain assuming that x is a random variable and that the function holds for {x|x E R }

That's an entirely different and unrelated question. Yes, the identity function on the reals is a continuous function. We still don't know what a continuous *set* is.

> But, suppose I defined the x−axis to be the set of rationals, not the set of reals (assuming it makes sense to do that − please inform me if that is an idiotic concept). If I were to graph the same function and said the domain was the set of rationals, would the function be considered a continuous function?

More generally, the identity function on any topological space is always continuous, if we are consistent in our choice of a topology. For example, we could take the identity function defined on the integers, and it would be a continuous function, even though there are huge gaps in the set of integers.

You can find a definition of what it means for a function to be continuous at <http://mathworld.wolfram.com/ContinuousFunction.html>. Notice that the definition doesn't say anything about "gaps". Also, there is nothing on that page that talks about continuous *sets*, only continuous *functions*.

> On the one hand, in the interval between 1.41421 and 1.41422, the point equal to the square root of 2 would be missing from such a graph, suggesting to me that it is not continuous. On the other hand, if the set of rationals is dense and if dense means that you can get infinitely close to the square root of 2 using rationals, then perhaps the graph is considered to be continuous. I simply have no idea one way or the other and I have no idea whether the concept is even sensible. But hopefully, it is now clear that I am trying to develop an intuitive understanding of what dense means and how it is or is not related to the concept of continuous, whether through functions, through sets or through some other thing. If those two concepts belong in two different arenas, that would be worth knowing as well.

I suspect the concept you are trying to get at is that of a connected set, rather than a "continuous" set. The reals are a connected set, but the rationals are not. See <http://mathworld.wolfram.com/ConnectedSet.html>.

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Dave Seaman
Oral Arguments in Mumia Abu−Jamal Case heard May 17
U.S. Court of Appeals, Third Circuit
<http://www.abu−jamal−news.com/>
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