Divide the joint pdf plane

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Say I have two independant random variables X and Y follow uniform distribution, with X~U(0,1) and Y~U(0,2), respectively. Hence, their joint pdf of X and Y is f(X,Y)=1/2 over a rectangular region on the XY plane (the four vertices are (0,0), (0,2), (1,2) and (1,0)). Now I want to divide the region to two parts S1 and S2 with same area, e.g., the straight line X=1/2 or Y=1 both satisfy this requirement. (the physical meaning behind this is that I want to find a scheme that chooses X and Y both with probability 1/2, and with the pdf being uniform in this simple example, the probability is equivalent to area). The objective of the division is to maximal the sum of the expectation of Y on S2 and the expectation of X on S1.

A more technical expression can be: given two independant r.v. X and Y, I want to design a selecting scheme such that I choose X and Y with equal probability. The reward I got is the value of the chosen r.v. Now the objective is to maximal the expectation of the reward.

Some natural extension of this problem can be:
(1). Other pdf distribution other than U;
(2). Choosing X and Y with unequal probability (p and 1−p);
(3). Choosing between multiple r.v. (N independant r.v.);
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