Re: Compact open topology

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- *From*: sanchopancho80@xxxxxx
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On 27 Jan., 21:09, Philippe Gaucher <p...@xxxxxxxxxxxxx> wrote:

> sanchopanch...@xxxxxx writes:
>
>> Yes, that's what I wanted to say: The TOP map hom(X x Y, Z) −−−> hom(Y, hom(X,Z)) does not EXIST in general in TOP, because it is not continuous, right?
>
> For a category C, I denote by C(X,Y) the set of morphisms from X to Y. So in Top, the set of continuous maps from X to Y is denoted by Top(X,Y). Of course, Top(X,Y) is included in Set(X,Y). And from now on, hom(X,Y) is the set Top(X,Y) equipped with the compact−open topology.
>
> The only set map which always exists is Top(XxY,Z)−−>Set(Y,Set(X,Z)). If X is locally compact, the image of the set map Top(XxY,Z)−−>Set(Y,Set(X,Z)) is included in Set(Y,Top(X,Z)), and even in Top(Y,Hom(X,Z)). In categorical language, Xx−:Top−−>Top admits Hom(X,−):Top−−>Top as right adjoint. So if X is locally compact, there is a natural set bijection Top(XxY,Z)=Top(Y,Hom(X,Z)).
>
> The next step is to prove that this set bijection induces a homeomorphism Hom(XxY,Z)=Hom(Y,Hom(X,Z)). I don't think that it is true in the category of general topological spaces (in fact I don't know whether it is true). If both X and Y are locally compact, then XxY is locally compact. And then, here is a proof: pick a topological space T. Then Top(T,Hom(XxY,Z)) = Top(TxXxY,Z) = Top(TxX,Hom(Y,Z)) = Top(T,Hom(X,Hom(Y,Z))). Apply the Yoneda Lemma: one obtains the homeomorphism Hom(XxY,Z) = Hom(X,Hom(Y,Z)).
>
> pg.

Ok, I agree, thanks. The last thing about the homeomorphism is also true if X and Y are Hausdorff and only X locally compact I think, but this isn't so interesting for me. Thank you for helping me to understand this stuff better.
Sancho

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