Re: compactness in angels/devil problem

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- *From*: pauldepstein@xxxxxxx
- *Date*: Sat, 2 Feb 2008 21:45:49 −0800 (PST)

On Feb 2, 11:24 pm, David C. Ullrich <dullr...@xxxxxxxxxxx> wrote:

> On Fri, 1 Feb 2008 18:06:29 −0800 (PST), pauldepst...@xxxxxxx wrote:
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>> On Feb 1, 10:06 pm, David C. Ullrich <dullr...@xxxxxxxxxxx> wrote:
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>>> On Fri, 1 Feb 2008 01:29:08 −0800 (PST),
>>> pauldepst...@xxxxxxx wrote:
>>>
>>>> The literature on the
>>>> angels−and−devilproblem often refers to a
>>>> "compactness argument" for passing from
>>>> conclusions about finite
>>>> boards to conclusions about the infinite case.
>>>> What is this
>>>> "compactness argument" and which
>>>> topology is the compactness concept
>>>> being applied to?
>>>
>>> What's the angels−and−devils problem?
>>>
>>> Quite possibly the compactness being referred to is from
>>> logic: If S is a collection of formulas and every finite subset
>>> of S has a model then S has a model.
>>>
>>> I could tell you what compact toplogy that's connected
>>> with, at least in the case of propositional logic, but it
>>> will take a little space. So first tell me what the d/a problem
>>> is and what sort of assertions about the problem you're
>>> talking about − the theorem I have in mind could be
>>> irrelevant.

Thank you,

Paul Epstein

David C. Ullrich

Hi David,

Informal version of angels/deviltheory: The angel is on a square chessboard which is infinite in all directions (Z x Z in other words). For each square of the chessboard, there is a finite set of squares which the angel can visit on the next move (you get a different problem or question for each rule determining the finite set). Thedeviland the angel take alternate moves. Thedevilmoves by eating one non−occupied square and therefore preventing the angel landing on it. Can thedevilrun the angel out of moves? (As stands, not a meaningful problem because I haven't given a rule determining the angel's possible moves.)

I'm not sure − I think the idea is that there are no constraints on thedevil'smove, on any turn he can eat any square he wants?
I'll assume yes until you say.

It's not clear to me right away whether the Compactness Theorem from logic is relevant, although I suspect one could set something up so it is. It does seem like there's some sort of compactness that _is_ relevant.

But: You've answered my first question but not the second:
What sort of assertion do people make, that they claim follows for an infinite board because it's true for every finite board?

The reason I ask is this: It's clear that, for example, "thedevil always wins" is _always_ true on any finite board (unless I'm misunderstanding the rules): Thedevileats a new square each move, and at some point there are no uneaten squares left. But it's not clear that thedevilalways wins on any infinite board (and for example if the queen could move as in chess, which of course contradicts the fact that she can only move to finitely many squares, then it's clear that thedevilnever wins on an infinite board.)

So my best guess as to what sort of assertion you're
talking about makes no sense, hence my confusion:
My best guess would be "If the queen can win on
any finite board then the queen can win on an
infinite board", but that makes no sense really since
the queen can _never_ win on a finite board. Similarly
for other guesses.

Yet another question: Is the queen's rule "translation−
invariant"? I suspect the answer's yes because that's the
only sort of rule you had in mind, but it seems like it
may be important so we need to know for sure.

What I mean by the question: A translation−invariant
rule would be defined by a fixed set of offsets from the
current position, for example "two squares up, or
one up and two to the right, or one down and one left"
would be a translation−invariant rule.

While for example "move like a king if you're
currently on a white square, move like a knight
if you're currently on a black square" would be
a rule that's not translation−invariant.

I have good guesses for the answers to most of my
question, except for the one about what sort
of assertion we're talking about...

> Concrete example: Suppose the angel moves like a king in the version of chess played in the US, England, Eastern Europe and elsewhere (in fact, the most globally widespread version of chess.) Then thedevil can trap the angel.
>
> Yes, David, thank you so much for your offer of explaining the related topological notion of compactness. I keenly await. (But don't neglect the backgammon.)
>
> Paul Epstein

David C. Ullrich− Hide quoted text −

− Show quoted text −

Thanks for your post. The devil does always win on a finite board, yes. Therefore an infinite board is always assumed to make the problem interesting. Translation−invariance also tends to be assumed in the literature I've read. Yes, you seem to be not misunderstanding the rules.

Example of a "compactness argument" in this context (apologies for omitting this from my previous post). Suppose that, for every N, the angel is able to travel a distance of >= N from the origin. Then, "by a compactness argument", we know that the angel can avoid being trapped by the devil.

Paul Epstein
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