

Algebra with finite field..

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Hello sir~

If F is a field of prime characteristic p ,
then $(a + b)^{p^n} = a + b$ for all a, b in F
and all positive integers n .

pf)

Let a, b in F .

Applying the binomial theorem to $(a + b)^p$,

we have

$$\begin{aligned}(a + b)^p &= a^p + (p).a^{p-1}.b + \{p(p-1)/2\}.a^{p-2}.b^2 \\ &+ \dots + p.a.b^{p-1} + b^p \\ &= a^p + 0.a^{p-1}.b + 0.a^{p-2}.b^2 + \dots + 0.a.b^{p-1} + b^p \\ &= a^p + b^p \\ &= a + b (***)\end{aligned}$$

Proceeding by induction on n ,

suppose that we have $(a + b)^{p^{(n-1)}} = a + b$.

Then $(a + b)^{p^n} = [(a + b)^{p^{(n-1)}}]^p = (a + b)^p = a + b$.

I can't understand (***)part.

Namely, $a^p = a$, $b^p = b$.

Why ?