Re: Algebra with finite field..

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- *From*: "Mariano Suárez−Alvarez" <mariano.suarezalvarez@xxxxxxxxx>
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On Feb 5, 1:53 pm, José Carlos Santos <jcsan...@xxxxxxxx> wrote:

> On 05−02−2008 15:23, Fatal wrote:
>
> > I can't understand (***)part.
> > Namely, a^p = a, b^p = b.
> > Why ?
>
> This is false in general.
> Indeed, if it where true, every element of F would be a root of the
> polynomial X^p−X, thus the cardinality of F should be p: this is not the
> case if n>1.
>
> On the other hand, it is true if the hypothesis "F is a field of prime
> characteristic _p_" is replaced by the stronger hypothesis "F is a field
> with _p_ elements".
>
> Best regards,
>
> Jose Carlos Santos

In any case, the original statement, that

if F is a field of characteristic p, n is a positive
integer and a and b are in F,then (a+b)^(p^n) = a + b

is false, since, by taking b = 0 would imply, this would
imply that for all a in F we have a^p = a, which is absurd
unless F has exactly p elements.

What is true is

(*) if F is a field of characteristic p, n is a positive
integer and a and b are in F,then (a+b)^(p^n) = a^(p^n) + b^(p^n)

which follows trivially from the simpler

if F is a field of characteristic p and a and b are in F,
then (a+b)^p = a^p + b^p

which in turn follows easily from Newton's formula and
the fact that the binomial number binom(p,k) is
divisible by p as soon as 0 < k < p.

(*) might be what mina_world was after...

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