Re: Understanding the concept of field map..

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- *From*: Martin Wanvik <martinw@xxxxxxxxxxxx>

- *Date*: Fri, 22 Feb 2008 04:13:09 EST

> Hi everyone,
>
> I was looking at the following article on Wikipedia:
>
> http://en.wikipedia.org/wiki/Linear_map
>
> In the following statement:
>
> "Let V and W be vector spaces over the same field K.
> A function f : V
> ’ W is said to be a linear map if for any two vectors
> x and y in V and
> any scalar a in K, the following two conditions are
> satisfied:"
>
> So, V and W define a Euclidean set of numbers, right?

I'd call them vectors, as they're elements of a vector space.

> Now what do they
> exactly mean when it says they are over the same
> field or for that
> matter what would it mean when they are over a
> different field?

I'm not entirely sure I know where exactly your confusion lies, but I'll try to explain a bit anyway:

I'm guessing you haven't studied abstract algebra, so it's quite natural that you're confused about the above definition. A vector space is a certain algebraic structure in which you can add two vectors together and multiply by a scalar; think of n−tuples of real numbers,
you add them in the following way:

(x_1, ..., x_n) + (y_1, ..., y_n) = (x_1 + y_1, ..., x_n + y_n)

and you can multiply a vector by another real number r
in the following way

r(x_1, ..., x_n) = (rx_1, rx_2, ..., rx_n)

The scalars form an algebraic structure of their own, called a field, in which you can add, subtract, multiply and divide according to the usual rules. Here, the scalars are real numbers (which is a field), and we would say that R^n (the set of n−tuples of real numbers) form a vector space over the field R.

So, in simple terms, vector spaces over the same field means that the scalars are the same. That is in fact needed in order for the definition of a linear map to make sense; L: V −−> W is linear if
1. L(v + v') = L(v) + L(v')
2. L(rv) = rL(v) (r is a scalar)

You see that in the second axiom, on the left hand side, we're first multiplying v by the scalar r (that happens in V) and then sending it over to W by the map L. On the right hand side, we send v over to W using L and then multiplying by the same scalar (that happens in W), so we need that scalar multiplication to make sense in W (which is why we require that both vector spaces have the same field of scalars; That requirement could be relaxed, one could instead demand that V is a vector space over K, W is a vector space over F with K a subfield of F)

For more precise definitions, see

http://en.wikipedia.org/wiki/Vector_space
http://en.wikipedia.org/wiki/Field_%28mathematics%29

> Also, I am a bit unsure about the terminology here:
>
> When they say, the dot or inner product is a map from
> R(n) X R(n) −−−> R
>
> Are they talking about the same map or are they
> saying that it is a
> transformation.

About the same map? Same as what? It means that the inner product takes a pair of vectors and produces a real number according to some specified rules.

> Also, in the above statement are we going from R(n)
> set of n tuples to
> R (set of all reals)?

No, from the set of _pairs of vectors_ to the reals.

Martin Wanvik

Sorry for these newbie questions....

Cheers,
Anja

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