

Re: Simple combinatoric problem

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- *From:* Robert Israel <israel@xx>
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richard@xxxxxxxxxxxxxxxx (Richard Tobin) writes:

In a shuffled pack of cards, what is the probability that an ace is adjacent to a king?

The answer (found by enumerating all the positions of the aces) is 284622747/585307450 or 48.63%, but can anyone find a simple way to show this?

You want the probability that at least one ace is adjacent to at least one king. Generalizing a bit, let $P(n, a, k)$ be the probability that at least one ace is adjacent to at least one king in a deck of n cards containing a aces and k kings, where $n \geq a + k$. Of course $P(n, a, k) = 0$ if $a = 0$ or $k = 0$. Let $Q(n, a, k)$ be the probability given that the top card is an ace, and $R(n, a, k)$ the probability given that the top card is a king. Then by conditioning on the top card we have

$$P(n, a, k) = a/n Q(n, a, k) + k/n R(n, a, k) + (n - a - k)/n P(n - 1, a, k)$$

and by conditioning on the second card

$$Q(n, a, k) = (a - 1)/(n - 1) Q(n - 1, a - 1, k) + k/(n - 1) + (n - a - k)/(n - 1) P(n - 2, a - 1, k)$$

$$R(n, a, k) = (k - 1)/(n - 1) R(n - 1, a, k - 1) + a/(n - 1) + (n - a - k)/(n - 1) P(n - 2, a, k - 1)$$

Now calculate...

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Robert Israel israel@xx
Department of Mathematics <http://www.math.ubc.ca/~israel>
University of British Columbia Vancouver, BC, Canada