# Re: Simple combinatoric problem

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In article <rbisrael.20080227232814$379a@xxxxxxxxxxxxxxxx>,
Robert Israel <israel@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx> wrote:

> richard@xxxxxxxxxxxxxxx (Richard Tobin) writes:
>
>> In a shuffled pack of cards, what is the probability that an ace
>> is adjacent to a king?
>>
>> The answer (found by enumerating all the positions of the aces) is
>> 284622747/585307450 or 48.63%, but can anyone find a simple way to
>> show this?
>
> You want the probability that at least one ace is adjacent to at least
> one king. Generalizing a bit, let P(n, a, k) be the probability that at
> least one ace is adjacent to at least one king in a deck of n cards
> containing a aces and k kings, where n >= a + k. Of course P(n,a,k) = 0
> if a = 0 or k = 0. Let Q(n, a, k) be the probability given that the top
> card is an ace, and R(n, a, k) the probability given that the top card is a
> king. Then by conditioning on the top card we have
>
> P(n,a,k) = a/n Q(n,a,k) + k/n R(n,a,k) + (n−a−k)/n P(n−1,a,k)
>
> and by conditioning on the second card
>
> Q(n,a,k) = (a−1)/(n−1) Q(n−1,a−1,k) + k/(n−1) + (n−a−k)/(n−1) P(n−2,a−1,k)
> R(n,a,k) = (k−1)/(n−1) R(n−1,a,k−1) + a/(n−1) + (n−a−k)/(n−1) P(n−2,a,k−1)
>
> Now calculate...

Alternatively, use inclusion−exclusion.

The event ace−of−spades adjacent king−of−hearts can happen in
51 x 2 x 50−factorial ways.

Similarly for each of the other 15 adjacency events, so we have

16 x 51 x 2 x 50−factorial.

Now you have to subtract all the ways that 2 of the 16 adjacency events can take place. E.g., (AS adj KH) and (AS adj KD) happens in 50 x 2 x 49−factorial ways; (AS adj KH) and (AD adj KC) happens in 52−choose−4 x 8 x 48−factorial ways, etc.

Then add back in all the ways 3 adjacency events can take place, etc. You can't have more than 7 adjacency events taking place, so you'll know when to stop.

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Gerry Myerson (gerry@xxxxxxxxxxxxxxx) (i −> u for email)
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