Re: find the limit of this quotient

Source: http://sci.tech−archive.net/Archive/sci.math/2008−03/msg00505.html

- *From*: taxxon1987@xxxxxxxxx
- *Date*: Tue, 4 Mar 2008 07:12:28 −0800 (PST)

On Feb 8, 11:47 pm, magidin@xxxxxxxxxxxxxxxxx (Arturo Magidin) wrote:

> In article
> <36986358−f991−493f−91d2−328a91c84824@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
> Randy Poe <poespam−trap@xxxxxxxxx> wrote:
>
>> On Feb 8, 11:14 pm, "Carl R." <solrac...@xxxxxxxxxxx> wrote:
>>
>>> Let a_n be a sequence of real numbers which converges to 0.
>>>
>>> Question: What are the possible limits which can take the
>>> quotient
>>> which consists of the sequence
>>> a_(n+1)/a_n when n tends to infinity?
>>>
>>> Days ago, Ken Pledger showed the limit is 1 when the
>>> sequence
>>> converges to a nonzero limit.
>>> But somewhere in the proof he used the fact that the limit is
>>> nonzero to define epsilon as |l|/4 * epsilon.
>>> But what happens when the original sequences converges to
>>> 0?
>>>
>>> First, I think we need two things: to prove the sequence
>>> converges
>>> and then find the value of the limit.
>>> How can you do this?
>>> I think the answer is 1 and is the only possible value of the
>>> limit
>>> of the quotient but how do you prove this?

> Counterexample:
>
> a_n = 1/n! −> 0
>
> a_(n+1)/a_n = n!/(n+1)! = 1/(n+1) −> 0
>
> It all depends on how fast a_n converges to 0.
>
> If a_n converges to 0, then the ratio must converge to a value <= 1.

In fact, of absolute value less than or equal to 1.

(If the absolute value is greater than 1, then |a_{n+1}|>|a_n| for all sufficiently large n, hence the sequence cannot converge to 0).

> But I think that the limit could be anything in [0,1]. In fact I suspect it could be anything in [−1,1].

You've exhibited one with limit 0; to get one with limit 1, take a_n = 1/n. To get one with limit −1, take a_n = (−1)^n/n. Then a_{n+1}/a_n = (−1)^{n+1}*n/(−1)^n*(n+1) = −n/(n+1), with limit −1.

And for any nonzero r, −1 < r < 1, let a_n = r^n. Then lim a_n = 0, and lim(a_{n+1}/a_n) = lim(r^{n+1}/r^n) = lim(r) = r.

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"It's not denial. I'm just very selective about
what I accept as reality."
−−− Calvin ("Calvin and Hobbes" by Bill Watterson)
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this is interesting. But just wondering if you can generalize the idea. As in can you always prove that if the sequence converges to zero (lim a_n=0), there will be a subsequence a_n_k such that a_n_(k

+1)/a_n_k=0?
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