

# Re: Probability of picking a positive rational number at random

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- *From:* "Ross A. Finlayson" <raf@xxxxxxxxxxxxxxxx>
  - *Date:* Thu, 13 Mar 2008 21:11:05 -0700 (PDT)
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On Mar 13, 2:55 pm, quasi <qu...@xxxxxxxx> wrote:

On Thu, 13 Mar 2008 17:20:59 -0500, quasi <qu...@xxxxxxxx> wrote:

On Thu, 13 Mar 2008 15:03:57 -0700 (PDT), S\_Pa...@xxxxxxxxxxxx wrote:

I am thinking that if i picked a random integer  $n \geq 1$  i could use this distribution:  
The probability of the exponent of its 2 prime factor=0 is  $1/2$   
The probability of the exponent of its 2 prime factor=1 is  $1/4$   
...  
The probability of the exponent of its 3 prime factor=0 is  $2/3$   
The probability of the exponent of its 3 prime factor=1 is  $2/9$   
The probability of the exponent of its 3 prime factor=2 is  $2/27$   
...  
The probability of the exponent of its n'th prime factor=k is  $(P_n-1)/P_n^k$   
Each prime factor would be independent of the others.

It's not a valid distribution since, for every natural number n, the probability that the result of your proposed experiment yields the value n is 0.

Any valid concept of "random" natural number amounts to choosing a distribution on  $\mathbb{N}$ . In other words, effectively it amounts to the choice of a sequence

$p_1, p_2, p_3, \dots$

of nonnegative real numbers such that

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$$p_1 + p_2 + p_3 + \dots = 1$$

Of course, the values of the sequence can't all be the same, hence there is no such thing as a uniform distribution on  $\mathbb{N}$ .

But clearly, there are uncountably many distinct non-uniform distributions.

So suppose you choose one of them and use that distribution twice, independently, once for the numerator, once for the denominator.

By symmetry, the probability of getting a rational greater than 1 is the same as the probability of getting a rational less than 1, although that common probability will be *less* than  $1/2$ , for the simple reason that there is a positive probability of getting a rational *equal* to 1. Of course, the actual value of the common probability will depend on the choice of  $p_1, p_2, p_3, \dots$

Similarly, by symmetry, the probability of getting a rational strictly between  $1/2$  and 1 would equal the probability of getting a rational strictly between 1 and 2, but once again, the actual value of that common probability will depend on the choice of  $p_1, p_2, p_3, \dots$

Bottom line --- the question is a naive one. The answer depends critically on the choice of distribution on  $\mathbb{N}$ , and moreover, there is no natural such choice which would qualify as a universal default.

quasi

A uniform probability distribution over the naturals would have that the sum of a constant infinitely many times would equal one. Now, that kind of notion is well reflected in that of arguments about the differential, that the sum of infinitely many (constant) infinitesimal width differential areas is a finite sum and exactly one. (The inexistence of that constant leads to nonmeasurable sets, in for example Vitali's argument.)

Arguments have been presented here as to why the existence of a uniform distribution over  $\mathbb{R}_{[0,1]}$  implies a uniform distribution over  $\mathbb{N}_{[0, \infty)}$ .

<http://groups.google.com/group/sci.math/msg/2e346cb26bc92f7d>

Asymptotically, a wide variety of distributions over the naturals go flat to uniformity, just as many peak to Dirac's delta.

Imagine a distribution over the rationals by selecting an integer at random (and from a population as large as to have the sample be uniform with respect to features of the model) and then selecting a

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rational, similarly uniformly, for example a censored uniformly random real from  $[0,1]$ , then the probability that said rational was in  $[0,1]$  would be the same probability that the integer part was zero.

Now, those quantities would be equal, but infinitesimal, but positive, and only commensurable to standard positive real quantities as being less than them and standard zero as being greater.

ZF is inconsistent.

Whatever natural one there would be would fulfill all its properties.

Ross

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