

# Re: Probability of picking a positive rational number at random

---

*Source:* <http://sci.tech-archive.net/Archive/sci.math/2008-03/msg02340.html>

---

- *From:* "Ross A. Finlayson" <raf@xxxxxxxxxxxxxxxx>
  - *Date:* Sat, 15 Mar 2008 21:51:23 -0700 (PDT)
- 

On Mar 15, 6:04 pm, Tim Little <t...@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx> wrote:

On 2008-03-15, Ross A. Finlayson <r...@xxxxxxxxxxxxxxxx> wrote:

So now you're going from a probability distribution over  $[0,1]$  defined by Lebesgue measure to a completely undefined probability distribution over the set of all well-orderings of distributions of natural numbers? How do you know that one with the properties you want even exists?

Can't you just use classical probabilities to show that?

Classical probability is more limited than measure-theoretic probability, so it can't help you. Can you even explain in terms of mathematical properties what you mean by your desired uniform distribution?

Simply asserting "for all  $x,y: P(\{x\}) = P(\{y\})$ " isn't enough, since there are infinitely many distributions satisfying that relation. Feel free to try to define a uniform distribution over all uniform distributions over all well-orderings of distributions over natural numbers. That won't get you any closer, and I'll feel free to call it ridiculous.

That iota is great among infinitesimals represents that summing infinitely many of them yields one.

Originally you said "greatest".

## Re: Probability of picking a positive rational number at random

In a theory like ZFC with well-orderable reals, and the ability to "sample" an element of the population of real numbers by infinitely many Bernoulli trials implying the ability to select an ordinal at uniform random from their initial ordinal, because well-orderings of the reals are so random, there is the ability to sample from the naturals a value such that for any other value, the probability of its selection is exactly the same, and the sum of their probabilities, their whole, in that they are mutually exclusive events one of which occurs, censored to countable, is one.

Apart from being a gross run-on sentence, this is rubbish.  
Well-orderings aren't random.

Simply, sample a real by infinitely many Bernoulli trials and discard it if a particular well-ordering has that real not mapping to a finite ordinal.

So your "algorithm" is: Generate a countably infinite number of binary digits. Do this (on average) \*uncountably\* many times, until you get a countable ordinal. I don't know what you're on, but it's not mathematics.

– Tim

About the suggestion of the utility of classical probabilities and analysis without regard to measure theoretic foundations, that was about proving those laws of probability that you deferred to measure theory. (That could have been quoted inline with the comment above, in responses I don't reply inline, which while easily indicating relevant context of response, interrupts.) Besides that measure theory isn't necessary for the reason you mentioned, there are as well nonstandard measure theories in consideration, besides nonstandard non-measure-theoretic foundations for analysis, where continuous, discrete, and mixed distributions can be analyzed without standard "measure theory", albeit using theories of measure. (There are other types of analysis than standard and Nonstandard. Some modern probabilities that aren't classical kind of defy measure theory as well, in for example parastatistics.)

About the iota comment, yeah I did say "greatest" and later equivocated for it to be that. That's rare. It was just to draw parallel in notion that where there's an infinity, that's in a way inaccessible from finity and counting, yet suitable as a prototype of infinite objects, that it's symmetrical in concept, and thus elegant and even natural, to consider that there's an infinitesimals, that's

## Re: Probability of picking a positive rational number at random

in a way inaccessible from finity and dividing, particularly where it would be effervescent differential, from infinitesimal analysis (integral calculus, analysis). You accept "smallest infinity", although infinities are defined often by having a smaller infinity, where's "greatest infinitesimal"?

It is ridiculous for you to talk about "a uniform distribution over all uniform distributions over all well-orderings of distributions over natural numbers." In well-ordering the distributions over natural numbers, in a bijection to the unit interval of reals, by selecting a real at random, each distribution has the same probability as any other to be selected, and it's not obvious that there's some way to correlate adjacency in reals to adjacency in the distributions over the naturals.

Apart from being a run-on sentence you didn't read it right. It seems understood: given a well-ordering of the reals, there exists for the reals mapping to finite ordinals a method to sample them, trans-finitely, by sampling the reals "until" one of them is a sample. (That's basically another well-ordering of the reals, in discarding duplicate samples, some least element of which map to a finite ordinal in the original's.) Because a well-ordering of the reals is so random (in terms of Kolmogorov complexity and  $\Pi_1^1$  or what it is) unless the reals are actually in a contiguous sequence the real indicating a finite ordinal has that the ordinal is exactly as likely as each other ordinal to be the sample, regardless of the development of initial segments of the path that is the real.

And: that very well satisfies the meaning of "uniformity".

(While it leads to obvious contradictions in some entrenched uses of those words doesn't affect that given a fair coin and transfinite induction a random integer is a probabilistic event, as one of the natural integers, quite uniform: constant, in its probability among the others.)

Asymptotically, a random integer from  $UNIF(0,N)$  doesn't have an expected value.

Where a natural integer is sampled, and the probability of each is a constant, that's uniform.

Ross

—

Finlayson Consulting

.