Re: Probability of picking a positive rational number at random

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- *From*: S_Paske@xxxxxxxxxxx
- *Date*: Mon, 17 Mar 2008 08:13:57 −0700 (PDT)

On Mar 16, 3:58 am, "Ross A. Finlayson" <r...@xxxxxxxxxxxxxxx> wrote:

> On Mar 15, 11:10 pm, Tim Little <t...@xxxxxxxxxxxxxxxxxxxxxxxxxxx>
> wrote:
>
> > On 2008−03−16, Ross A. Finlayson <r...@xxxxxxxxxxxxxxx> wrote:
> >
> > > (That could have been quoted inline with the comment
> > > above,
> > > in responses I don't reply inline, which while easily
> > > indicating
> > > relevant context of response, interrupts.)
> >
> > Ah, you prefer an uninterrupted monologue rather than a conversational
> > style. Got it.
> >
> > > It is ridiculous for you to talk about "a uniform distribution
> > > over
> > > all uniform distributions over all well−orderings of
> > > distributions
> > > over natural numbers."
> >
> > You're the one who said:
> > Then, using the distribution that happens to be marked by that
> > ordinal in a "random" well−ordering of the distributions of the

naturals

I had just noticed that in trying to get from a perfectly well defined uniform distribution over reals to one over naturals, you were requiring arbitrary well−orderings followed by random distributions over well−orderings. I was just saving you a step.

> In well−ordering the distributions over natural numbers, in a bijection to the unit interval of reals, by selecting a real at random, each distribution has the same probability as any other to
> be selected

Only in the sense that every distribution has probability zero.

The biggest problem with your proposal is that for infinitely many well−orderings (quite possibly all of them), the event of deriving any given value n corresponds to a non−measurable set in the reals. Hence its probability is meaningless.

> Apart from being a run−on sentence you didn't read it right.

That is probably because you didn't write it right. The quantity of grammatical errors in your 9−line sentence rendered it nearly unintelligible. Just what did you mean by the clause "because well−orderings of the reals are so random"?

It is entirely possible for a well−ordering of the reals and a well−ordering of the distributions over natural numbers to result in a 99.9% probability of selecting a distribution for which P(n=1) = 0.999.

Apparently to avoid that, you specified that the well−ordering should be "random". How do you pick a random well−ordering if you can't even pick a random natural number?

> > It seems understood: given a well−ordering of the reals, there
> > exists for the reals mapping to finite ordinals a method to
> > sample
> > them, trans− finitely
>
> There does not so exist, in any theory I've seen. Perhaps you can come up with a theory that can model such processes and isn't full of contradictions, but I doubt it.
>
> − Tim

By "random" well−ordering I meant "any."

I'm surprised that you describe the putative uniform distribution over reals "perfectly well defined." It's ridiculous for _you_ to say that because to use the terms they essentially mean what you say.

About the well−ordered reals (not arguing that they're naturally well−ordered, although it's been so stated) that correspond to finite ordinals being a non−measurable set, actually in measure theory they'd be a measure zero set because there are countably many. Basically, in the context of probability and a putative uniform distribution over the naturals, that constant, the existence of which would lead to a re−write of proofs establishing the existence of non−measurable sets in the first place i.e. re−Vitali−zation, that constant value would be infinitesimal, non−standard, and in general outside the semantic realm of standard measure theory. That doesn't deny what it is, only that standard measure theory is mute about it.

I'm not talking down to you, there are not any grammatical errors in that long sentence, the style is not so great. The described Bernoulli trials should be described as fair Bernoulli trials with equal probabilities of failure and success, and are implicitly so above.

Well−orderings of the reals are random because some of them map to nonrecursive ordinals (in ZFC). Absolutely no specification can reach past there, although for each ordinal through recursive ordinals, the readout of that initial segment is the specification. For each countable initial segment of a well−ordering of standard real numbers in ZFC, the measure is zero, they're not even everywhere dense for recursively many.

http://www.google.com/search?hl=en&q=real+numbers+complexity+Kolmogorovhttp://search.live.com/result

Now, I am wondering how it might be that the distributions could possibly be skewed thus that, for example, all the reals less than .999 would map to distributions with P(n = 1) =.999, where there are cardinally as many distributions of (over) the naturals with that as there are otherwise. That would have that all the reals less than .999 mapped to ordinals less than those of (.999,1), and that all the distributions with P(n=1)=.999 preceded all the others, and that they had the same order type as the initial segment of the well−ordering of the reals well−ordering [0, .999]. That may be so, "adjacency" as above in that sense, what it doesn't say is that for almost all well−orderings of reals and well−orderings of natural distributions, nothing at all like that would be the case.

Well, in a transfinite course of passage, in assigning random values from {0,1} to ordinals, from each limit ordinal onwards there is exactly one omega word.So, in terms of all the ordinal mappings, 2^alpha for ordinal alpha, how many limit ordinals are there in it? In omega^2 there are omega many limit ordinals, omega^3 there are omega^2 many limit ordinals, etcetera. Past epsilon is it clear how many limit ordinals there are? Anyways, there are ordinal enumerations with many or all the reals therein encoded the same, for example constant 0. By the same token, there are those ordinal enumerations that encode each real as an infinite binary sequence, in the same cardinality as of reals as singletons (unless it took all the ordinals equivalent to the reals to well−order them, as opposed to just all the ordinals less than them).

So, of those structures, well−orderings of multisets of infinite binary strings, censored to uniqueness, appended with others until there is a least element mapping to a finite ordinal, there is nothing about the structure that says anything about which finite ordinal is the image of the least element, except that any other real that maps to a finite ordinal is equally as likely to be that random infinite bit sequence.

Here's another way to look at it, at least up through recursive ordinals. Each of those ordinals is basically of the form (in reversed ordinal arithmetical evaluation) finite ordinal n_0 plus n_1 omega^1 + n_2 omega^2 + ... So, the first real "generated" by flipping fair coins that maps to a recursive ordinal indicates a finite ordinal at random, and, at uniform random in that nothing can be said about it except that it's just as likely as any other.

In a transfinite course of passage, up past ordinals equivalent to the reals (in ZFC), almost all of those well−orderings of multisets would have more than countably many different reals in the multiset, then due to uniformity in distribution, one of those would likely map to a recursive ordinal.

Of course, another method was described early in the development of

> this thread, which began about positive rational numbers at random, and is now about how given a natural integer, and another natural integer with no other information about either, that the rational gambler would wager a dollar to win one that they were coprime, because the probability of those two numbers being coprime is greater than one half. When nothing can be said about an integer except that it's an integer, then its probability of being 0 mod 2, 3, ..., n is 1/n, and so is its probability of being 1, 2, ..., n−1. The concept of probability, its meaning, has that for two anonymous distributions of the integers, the probability of two independent samples being coprime is 6/pi^2, that any kind of predictive power based on knowledge of characteristics that apply to all of the population of all the natural integers is of a uniform population, closed to subtraction each merely, and exactly, one different from the previous and next, as Spinoza put them, naturally a continuum.
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> Ross
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> −−
> Finlayson Consulting− Hide quoted text −
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Ross,

The question is if given a 'random rational r' in this way, what is the probability that it is less than 1? How about < 2? Picking random integers it is difficult impossible to know anything about its magnitude. However, if one replaces the 'random' integer with a 'random' rational, it appears to me, it can be bounded. So for example, p(r<2)=p(r>2) and p(r between 1/2 and 2)=?

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