Re: Probability of picking a positive rational number at random

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- *From*: S_Paske@xxxxxxxxxxx
- *Date*: Mon, 17 Mar 2008 16:54:13 −0700 (PDT)

On Mar 17, 4:18 pm, "Ross A. Finlayson" <r...@xxxxxxxxxxxxxxx> wrote:

> On Mar 17, 7:13 am, S_Pa...@xxxxxxxxxxx wrote:
>
> > On Mar 16, 3:58 am, "Ross A. Finlayson" <r...@xxxxxxxxxxxxxxx> wrote:
> >
> > > On Mar 15, 11:10 pm, Tim Little
> > > <t...@xxxxxxxxxxxxxxxxxxxxxxxxxxx>
> > > wrote:
> > >
> > > > On 2008−03−16, Ross A. Finlayson
> > > > <r...@xxxxxxxxxxxxxxx> wrote:
> > > >
> > > > > (That could have been
> > > > > quoted inline with the
> > > > > comment above,
> > > > > in responses I don't reply
> > > > > inline, which while easily
> > > > > indicating
> > > > > relevant context of response,
> > > > > interrupts.)
> > > >
> > > > Ah, you prefer an uninterrupted monologue
> > > > rather than a conversational
> > > > style. Got it.

> It is ridiculous for you to
> talk about "a uniform
> distribution over
> all uniform distributions
> over all well−orderings of
> distributions
> over natural numbers."

You're the one who said:
  Then, using the distribution that happens to be marked by that
  ordinal in a "random" well−ordering of the distributions of the
  naturals

I had just noticed that in trying to get from a perfectly well defined
uniform distribution over reals to one over naturals, you were
requiring arbitrary well−orderings followed by random distributions
over well−orderings. I was just saving you a step.

> In well−ordering the
> distributions over natural
> numbers, in a
> bijection to the unit interval
> of reals, by selecting a real
> at
> random, each distribution
> has the same probability as
> any other to
> be selected

Only in the sense that every distribution has probability zero.

The biggest problem with your proposal is that for infinitely many
well−orderings (quite possibly all of them),

the event of deriving any
given value n corresponds to a
non−measurable set in the reals. Hence
its probability is meaningless.

> Apart from being a run−on
> sentence you didn't read it
> right.

That is probably because you didn't write it
right. The quantity of
grammatical errors in your 9−line sentence
rendered it nearly
unintelligible. Just what did you mean by
the clause "because
well−orderings of the reals are so random"?

It is entirely possible for a well−ordering of
the reals and a
well−ordering of the distributions over
natural numbers to result in a
99.9% probability of selecting a distribution
for which P(n=1) = 0.999.

Apparently to avoid that, you specified that
the well−ordering should
be "random". How do you pick a random
well−ordering if you can't even
pick a random natural number?

> It seems understood: given a
> well−ordering of the reals,
> there
> exists for the reals mapping
> to finite ordinals a method
> to sample
> them, trans− finitely

There does not so exist, in any theory I've
seen. Perhaps you can
come up with a theory that can model such
processes and isn't full of
contradictions, but I doubt it.

− Tim

By "random" well−ordering I meant "any."

I'm surprised that you describe the putative uniform
distribution over
reals "perfectly well defined." It's ridiculous for _you_ to
say that
because to use the terms they essentially mean what you say.

About the well−ordered reals (not arguing that they're
naturally well−
ordered, although it's been so stated) that correspond to finite
ordinals being a non−measurable set, actually in measure
theory they'd
be a measure zero set because there are countably many.
Basically, in
the context of probability and a putative uniform distribution
over
the naturals, that constant, the existence of which would lead
to a re−
write of proofs establishing the existence of non−measurable
sets in
the first place i.e. re−Vitali−zation, that constant value would
be
infinitesimal, non−standard, and in general outside the
semantic realm
of standard measure theory. That doesn't deny what it is,
only that
standard measure theory is mute about it.

I'm not talking down to you, there are not any grammatical
errors in
that long sentence, the style is not so great. The described
Bernoulli trials should be described as fair Bernoulli trials

with
equal probabilities of failure and success, and are implicitly
so
above.

Well−orderings of the reals are random because some of
them map to
nonrecursive ordinals (in ZFC). Absolutely no specification
can reach
past there, although for each ordinal through recursive
ordinals, the
readout of that initial segment is the specification. For each
countable initial segment of a well−ordering of standard real
numbers
in ZFC, the measure is zero, they're not even everywhere
dense for
recursively many.

http://www.google.com/search?hl=en&q=real+numbers+complexity+Kolmogor...

Now, I am wondering how it might be that the distributions
could
possibly be skewed thus that, for example, all the reals less
than .
999 would map to distributions with P(n = 1) =.999, where
there are
cardinally as many distributions of (over) the naturals with
that as
there are otherwise. That would have that all the reals less
than .
999 mapped to ordinals less than those of (.999,1), and that
all the
distributions with P(n=1)=.999 preceded all the others, and
that they
had the same order type as the initial segment of the
well−ordering of
the reals well−ordering [0, .999]. That may be so,
"adjacency" as
above in that sense, what it doesn't say is that for almost all
well−
orderings of reals and well−orderings of natural distributions,
nothing at all like that would be the case.

Well, in a transfinite course of passage, in assigning random values
from {0,1} to ordinals, from each limit ordinal onwards there is
exactly one omega word.So, in terms of all the ordinal mappings,
2^alpha for ordinal alpha, how many limit ordinals are there in it?
In omega^2 there are omega many limit ordinals, omega^3 there are
omega^2 many limit ordinals, etcetera. Past epsilon is it clear how
many limit ordinals there are? Anyways, there are ordinal
enumerations with many or all the reals therein encoded the same, for
example constant 0. By the same token, there are those ordinal
enumerations that encode each real as an infinite binary sequence, in
the same cardinality as of reals as singletons (unless it took all the
ordinals equivalent to the reals to well−order them, as opposed to
just all the ordinals less than them).

So, of those structures, well−orderings of multisets of infinite
binary strings, censored to uniqueness, appended with others until
there is a least element mapping to a finite ordinal, there is nothing
about the structure that says anything about which finite ordinal is
the image of the least element, except that any other real that maps
to a finite ordinal is equally as likely to be that random infinite
bit sequence.

Here's another way to look at it, at least up through recursive
ordinals. Each of those ordinals is basically of the form (in
reversed ordinal arithmetical evaluation) finite ordinal n_0 plus n_1
omega^1 + n_2 omega^2 + ... So, the first real "generated" by
flipping fair coins that maps to a recursive ordinal indicates a
finite ordinal at random, and, at uniform random in that nothing can

be said about it except that it's just as likely as any other.

In a transfinite course of passage, up past ordinals equivalent to the reals (in ZFC), almost all of those well−orderings of multisets would have more than countably many different reals in the multiset, then due to uniformity in distribution, one of those would likely map to a recursive ordinal.

Of course, another method was described early in the development of this thread, which began about positive rational numbers at random, and is now about how given a natural integer, and another natural integer with no other information about either, that the rational gambler would wager a dollar to win one that they were coprime, because the probability of those two numbers being coprime is greater than one half. When nothing can be said about an integer except that it's an integer, then its probability of being 0 mod 2, 3, ..., n is 1/ n, and so is its probability of being 1, 2, ..., n−1. The concept of probability, its meaning, has that for two anonymous distributions of the integers, the probability of two independent samples being coprime is 6/pi^2, that any kind of predictive power based on knowledge of characteristics that apply to all of the population of all the natural integers is of a uniform population, closed to subtraction each merely, and exactly, one different from the previous and next, as Spinoza put them, naturally a continuum.

Ross

−−
Finlayson Consulting− Hide quoted text −

− Show quoted text −

Ross,

The question is if given a 'random rational r' in this way, what is the probability that it is less than 1? How about < 2? Picking random integers it is difficult impossible to know anything about its magnitude. However, if one replaces the 'random' integer with a 'random' rational, it appears to me, it can be bounded. So for example, p(r<2)=p(r>2) and p(r between 1/2 and 2)=?

Hi,

I don't know. It seems that it varies about other features of the system, basically axiomatized or proscriptive. In some cases it might be reasonable that given two ordered positive integer values n and d that half the time n > d, otherwise n <= d, and so half of the positive rationals are less than one and half greater. Yet, by another notion, only infinitesimally many of the rationals are between zero and one, exactly as many as of the naturals are zero, in terms of the "fixed proportion" of reals in the unit interval to reals of the positive real number line.

I think that the probability that a "random" rational falls within a given subset of the reals is that subset's density in the reals, where the rationals are "equidistributed" through the reals. The properties of measure of set within superset, in terms of describing a population by its elements, imply real measure of subsets of the reals, but not, necessarily, standard real measure.

Then, among the rational numbers, if they're equidistributed throughout the real number line, which they are, then the probability that a "random" rational is greater or less than zero is approximately one half. Yet, so is P(q < 1) and P(q < 2) and P(q < n) for finite n, "approximately" one half of the numbers are to the left and half to the right on any point on the line. However, where there might be no real (read finite) difference between P(q<1), P(q<2), etcetera, there is still that P(q<1) < P(q<2) < ... < P(q<n) < P(q<n+) < ..., ordered by the total ordering of the real numbers and the naturals as a subset

> of them.
>
> If there were a uniform distribution of the naturals, then EF, not a real function, would be its CDF, and Cantor's first/nested intervals doesn't apply to it, and the antidiagonal is at the end of the list, which is among reasons why I call it the natural/unit equivalency function. Also, there wouldn't be non−measurable sets, and there would be infinitesimals in the real numbers.
>
> Ross
>
> −−
> Finlayson Consulting

I understand your thoughts of rationals distributed among reals equally, but many of the conversations here i can not. I also understand why questions like these are hated so much here.

Let me describe the process of picking a random integer vs random rational and you will see my thinking, though obviously if i had any results i wouldn't be here asking someone else for a closed form. I know that an infinite product is not always a rational so maybe i should not even talk about rationals.

```
Result = 1
For k = 1 to oo
pk = the k'th prime
real = random real between 0 and 1 inclusive
sign = random real between 0 and 1 inclusive
sum = real
exponent=0
do while sum < real
sum += p−1/p^k
exponent+=1
end
**leave this out for integers, include for rationals**
if sign<1/2 exponent=−exponent
**
Result = Result * p^exponent
Print Result
end
```

Now, if one is choosing a random integer, in laymans terms: one is likely to get oo with this program. However, by allowing the exponent to be either positive or negative, one is likely to get oo/oo.

I am thinking the ratio converges for rationals in specified intervals because for every large prime in the numerator, it tends to be cancelled by the denominator. It is still impossible to pick a particular rational at random because p(n)−>0 at oo

Of course infinite products are not rationals, but this program produces rational results because the number of primes i use is finite. Still, if one were able to run this with some super machine, would it produce a definite real? If so, it seems to me that the most likely result is $\pm 1/1$. After all, the finite density function shows $p(1)>p(2)>p(3)$, of course at the limit this doesn't hold as $p(n)=0$, but does this imply $p(-1<r<1)$ diverges?
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