Re: isometries, and symmetry groups

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On Mar 30, 10:46 pm, Narcoleptic Insomniac
<i_have_narcoleptic_insom...@xxxxxxxxx> wrote:

> On Mar 30, 2008 7:43 PM CT, crossedproduct wrote:
>
>> If M is a (nonempty) set, we can consider the group
>> of all bijections M −> M.
>>
>> Suppose X is a subset of M; does the set of all
>> bijections which leave X invariant, necessarily form
>> a group ?
>> (A bijection f : M −> M which leaves X invariant is
>> such that f(X) = X).
>>
>> I'm pretty certain that it does.
>
> I believe that a few more conditions are required to
> gaurantee a group structure.

Given a set M and a subset X of M, the set
of all bijections f: M −−> M such that f(X) = X
is indeed a subgroup of the group S(X) of all
bijections X −−> X: it is closed under composition
and inversion, and it is not empty.

> [... snip ...]
>
>> How does this differ from the symmetry group of X,
>> which consists of all *isometries* of M (when M is a

> > metric space) leaving X fixed ?

They are very different: just consider pretty much
any example.

> It would seem that the difference lies in the fact that
> Homeo(X) is a group regardless of the topological space X
> being metric, whereas Isom(M) requires M be a metric
> space −− it would appear that Homeo(X) is a little more
> general than Isom(M).
>
> Of course, that begs the question: Is Homeo(M) =~ Isom(M)
> for every metric space M? Intuitively, I would like to
> say yes, however, this matter is a little beyond me at
> the moment.

Those two groups are not isomorphic in general.

Take for example a solid triangle T in the plane
whose all three sides are of different length:
then the only isometry T −−> T is the identity,
but the group of auto−homeomorphisms T −−> T is
huge.

> > In other words, why are isometries important to
> > studying (or defining, for that matter) groups of
> > transformations of plane figures, and not just those
> > bijections which leave them invariant?

Isometries are important when you are studying the
metric properties of a plane figure. If you are studying
other types of properties, then other groups become
interesting, in general.

By the way, you say `not just those bijections which leave
them invariant' but in general there will be many many many
more bijections leaving it invariant than isometries
of the figure...

−− m
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