Re: Entire function

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On 31−03−2008 13:39, TimmyTimTim wrote:

> Let f be entire such that f(z) is real when z is real, and f(z)
> is imaginary when z is imaginary. Prove that f(−z)=−f(z).

You can write f(z) as sum_n a_n z^n. Since a_n = f^{(n)}(0)/n! and
since _f_ maps real numbers into real numbers, every a_n is real. Now,
define _g_ as he odd part of _f_ and _h_ as its even part. If you prove
that f = g, then your problem is solved. This is equivalent to the
assertion h = 0. Let z be a purely imaginary complex number. Then h(z)
is real. But h(z) = f(z) − g(z), which is purely imaginary. So,
h(z) = 0. This proves that the restriction of _h_ to the set of purely
imaginary complex numbers is null. But _h_ is an integer function and
therefore _h_ is null everywhere.

Best regards,

Jose Carlos Santos
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