Re: Probability with bus..

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- *Date*: Mon, 31 Mar 2008 08:17:46 −0700 (PDT)

On Mar 31, 4:41 am, "mina_world" <mina_wo...@xxxxxxxxxxx> wrote:

> Hello teacher~
>
> A passenger arrives at a bus−stop at some arbitrary point in time.
> Buses arrive according to a uniform distribution on [0,1]. (Namely, per 1
> min.)
>
> What is the mean waiting time until the next bus ?
>
> _____
> Sorry. I need your advice.

Arrival of buses is a "renewal process" with a uniform inter−arrival time X. If you arrive after many buses have already gone by (that is, at a large time), the waiting−time distribution is, essentially, the limiting waiting−time distribution, whose probability density is g(x) = G(x)/EX, where G(x) is the complementary cumulative distribution of X and EX is the mean. We have H(x) = 1−x and EX = 1/2, so the density is g(x) = 2*(1−x) for 0 <= x <= 1, and g(x) = 0 for x < 0 or x > 1. The expected waiting time is int(x*g(x),x=0..1) = 2*int(x*(1−x),x=0..1) = 2*(1/6) = 1/3. Of course, this is larger than the "naive" value that you might get if you simply reason that you come, on average, halfway between two buses, and buses arrive spaced 1/2 minute on average, so you ought 1/4 minute on average. That would be wrong, but the reasons are subtle.

You might also be interested in the expected time Y between the two buses that "straddle" your arrival at the bus stop; that is, the time between the last bus before you come and the next bus after you come. The (limiting, or equilibrium) density of Y is g(y) = y*f(y)/EX, where f = inter−arrival time density. The mean is EY = E(X^2)/E(X), which gives EY = 2/3 for X~U(0,1). Again, this is larger than the average time EX = 1/2 minute between two buses, even though it IS the time between two specific buses−−−namely, the one that came before you and the one that comes after you.

If your arrival time is not large, you need to solve the renewal

process integral equation to get the waiting time density and expected waiting time.

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