Re: math −− finite union of rectangular regions

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- *From*: quasi <quasi@xxxxxxxx>
- *Date*: Tue, 01 Apr 2008 02:48:44 −0500

On Sun, 30 Mar 2008 23:24:36 −0700 (PDT), Mariano Suárez−Alvarez
<mariano.suarezalvarez@xxxxxxxxx> wrote:

> Let A be a finite set of closed rectangles in the plane
> whose sides are parallel to the coordinate axes whose
> union is simply connected. Let us assume that
>
> (*) for all R in A, the union of the rectangles
> in A − { R } is not simply connected.
>
> For each rectangle R in A, let
>
> C(R) = R − union { R' in A : R' != R }
>
> Let us call each of the connected components of C(R) such
> that its closure is contained in the interior of R a *hole*
> of A, and let us say that it belongs to R.

Yes, I follow.

> A hole of A belongs to exactly one rectangle in R, which is
> the only one which contains it. The holes are moreover disjoint.

Yes.

> Conversely, our hypothesis (*) is equivalent to the
> statement that to each rectangle in R belongs at least one
> hole.
>
> For each bounded subset X of the plane, let us call the sup
> of the y coordinates of the points in X the *height* of X.

Fine.

> Let as pick a hole H of minimal height among the
> holes of A. The boundary B of H is a finite polygonal
> closed arc, whose segments are parallel to the coordinate
> axes. Pick one horizontal segment in B which has minimal
> height among the horizontal segments of B. This segment
> shares a subsegment of positive length with the boundary
> of some rectangle S in A. Let H' be one of the holes in S.
> It is clear that the height of H' is strictly less that the
> weight of H. This is absurd.

Yes!

(although I assume you meant "height of H").

Kind of a "descent argument". If there's a hole, there has to be a lower one.

Very nice.

quasi
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