

## Re: Is the empty set a number?

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- *From:* "G.E. Ivey" <[george.ivey@xxxxxxxxxxxxxx](mailto:george.ivey@xxxxxxxxxxxxxx)>
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Do you understand the concept of "equivalence"? one way of defining the natural numbers is to set "0" to be the empty set, {}, "1" to be the set whose only member is the empty set, {}, "2" to be the set whose only members are the empty set and {} – that is, whose only members are 0 and 1 – {0, 1}, etc.

We can then define the "successor" of any number,  $x$ , to be the set containing  $x$  and all of its members and show that Peano's axioms for the natural numbers hold.

We could define addition of two such things by " $x + 0 = x$  and, (if  $b$  is not 1, then  $b = s(c)$  for some some  $c$ )  $x + b = s(x + c)$  when  $b$  is not 0". We could define multiplication of two such things by " $x * 0 = 0$ , and  $x * b = x + x * c$  for  $b$  not 0".

For that particular system, with those operations, yes, the empty set IS the number 0. But there are many other ways to define "numbers" that do not use sets as numbers. The important point is that they are all "equivalent" – they all give the same results. You can think of "number" in terms of any one of them.

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