Re: Ten points in a square

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- *From*: Angus Rodgers <twirlip@xxxxxxxxxxx>
- *Date*: Sat, 12 Apr 2008 01:42:30 +0100

On Fri, 11 Apr 2008 19:54:21 −0500, quasi <quasi@xxxxxxxx> wrote:

> On 12 Apr 2008 02:29:17 +0300, Phil Carmody
> <thefatphil_demunged@xxxxxxxxxxx> wrote:
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>> quasi <quasi@xxxxxxxx> writes:
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>>> On 11 Apr 2008 21:54:31 GMT, David W. Cantrell
>>> <DWCantrell@xxxxxxxxxxx> wrote:
>>>
>>>> "Zdislav V. Kovarik"
>>>> <kovarik@xxxxxxxxxxx> wrote:
>>>>
>>>>> It is a standard exercise on
>>>>> Pigeonhole Principle to
>>>>> prove:
>>>>>
>>>>> If you place 10 distinct
>>>>> points in a square of side 1,
>>>>> then at least
>>>>> two points will have
>>>>> distance no more than
>>>>> sqrt(2)/3 (about 0.4714).
>>>>>
>>>>> My question: This number
>>>>> is an upper bound for the
>>>>> minimum
>>>>> positive distance. Has
>>>>> anyone found the least
>>>>> upper bound?
>>>>>
>>>>> (It is at least 1/3, just place
>>>>> the points at lattice points
>>>>> with stepsize 1/3. With
>>>>> slightly more effort, one can
>>>>> replace

1/3 by sqrt(2)/(2*sqrt(2)+1),
about 0.3694.)

By the way, tens of millions
of pseudorandom
experiments have
not exceeded 0.32.

It is 0.421..., which follows from the packing
of ten unit circles, proven
optimal, shown at
<http://www.stetson.edu/~efriedma/cirinsqu/>.

It's not clear to me that the 10 circle packing configuration
answers
the OP's question.

There's no requirement for the placed points to be any given
distance
away from the boundary of the square.

Assuming that there's one circle abutting each edge of the
square, then you can be sure that all the centres fit within
a square with width 2 less, 4.747+. Then scale that down to
be 1, and count the distance between points as that of 2
radii. If you could improve on this ratio, then by adding
the 1−unit border back round the outside you'd be able to
improve on the packing found at DC's link.

I don't see it.

If you start with a maximal 10−circle packing configuration with all
equal circles of radius r, that induces a 10−point configuration with
min distance d = 2r.

The converse doesn't seem to automatic, and may not be true.

In other words, if you start with a 10−point configuration with min
distance d, that doesn't necessarily induce a 10−circle packing
configuration with all circles of radius r, where r = d/2. The issue
is the "outer" points, which need not be d away from the boundary of
the square.

It's late at night here, and I may be missing your point, but
it seems as if you're repeating the argument you gave earlier,
and forgetting to, as Phil put it, "scale down". If you start

with a 10−point configuration in a unit square, with minimum
separation d = 2r, then you have a packing of 10 circles into
a square of side at most 1 + 2r. So if d > .4213+, then when
the packing figure is scaled up, so that the circles all have
radius 1, the side of the square the circles are all packed
into will have side at most (1 + 2r)/r = 2 + 1/r < 6.474− −
which we know to be impossible. (I'm sorry, I haven't put this
very elegantly, and who knows, I may have reversed an inequal−
ity sign somewhere, and it may all be nonsense.)

−−
Angus Rodgers
(twirlip@ eats spam; reply to angusrod@)
Contains mild peril
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