

# Re: Ten points in a square

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- *From:* quasi <quasi@xxxxxxxx>
  - *Date:* Fri, 11 Apr 2008 21:05:04 -0500
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On Sat, 12 Apr 2008 01:42:30 +0100, Angus Rodgers  
<twirlip@xxxxxxxxxxxx> wrote:

On Fri, 11 Apr 2008 19:54:21 -0500, quasi <quasi@xxxxxxxx> wrote:

On 12 Apr 2008 02:29:17 +0300, Phil Carmody  
<thefatphil\_demunged@xxxxxxxxxxxx> wrote:

quasi <quasi@xxxxxxxx> writes:

On 11 Apr 2008 21:54:31 GMT, David W.  
Cantrell  
<DWCantrell@xxxxxxxxxxxx> wrote:

"Zdislav V. Kovarik"  
<kovarik@xxxxxxxxxxxx>  
wrote:

It is a  
standard  
exercise on  
Pigeonhole  
Principle to  
prove:

If you place  
10 distinct  
points in a  
square of  
side 1, then  
at least  
two points  
will have  
distance no  
more than

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$\sqrt{2}/3$   
(about  
0.4714).

My  
question:  
This  
number is  
an upper  
bound for  
the  
minimum  
positive  
distance.  
Has anyone  
found the  
least upper  
bound?

(It is at least  
 $1/3$ , just  
place the  
points at  
lattice  
points  
with  
stepsize  $1/3$ .  
With  
slightly  
more effort,  
one can  
replace  
 $1/3$  by  
 $\sqrt{2}/(2*\sqrt{2}+1)$ ,  
about  
0.3694.)

By the way,  
tens of  
millions of  
pseudorandom  
experiments  
have  
not  
exceeded  
0.32.

It is 0.421..., which follows  
from the packing of ten unit  
circles, proven

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optimal, shown at  
<<http://www.stetson.edu/~efriedma/cirinsqu/>>.

It's not clear to me that the 10 circle packing configuration answers the OP's question.

There's no requirement for the placed points to be any given distance away from the boundary of the square.

Assuming that there's one circle abutting each edge of the square, then you can be sure that all the centres fit within a square with width 2 less, 4.747+. Then scale that down to be 1, and count the distance between points as that of 2 radii. If you could improve on this ratio, then by adding the 1-unit border back round the outside you'd be able to improve on the packing found at DC's link.

I don't see it.

If you start with a maximal 10-circle packing configuration with all equal circles of radius  $r$ , that induces a 10-point configuration with min distance  $d = 2r$ .

The converse doesn't seem to automatic, and may not be true.

In other words, if you start with a 10-point configuration with min distance  $d$ , that doesn't necessarily induce a 10-circle packing configuration with all circles of radius  $r$ , where  $r = d/2$ . The issue is the "outer" points, which need not be  $d$  away from the boundary of the square.

It's late at night here, and I may be missing your point, but it seems as if you're repeating the argument you gave earlier, and forgetting to, as Phil put it, "scale down". If you start with a 10-point configuration in a unit square, with minimum separation  $d = 2r$ , then you have a packing of 10 circles into a square of side at most  $1 + 2r$ . So if  $d > .4213+$ , then when the packing figure is scaled up, so that the circles all have radius 1, the side of the square the circles are all packed into will have side at most  $(1 + 2r)/r = 2 + 1/r < 6.474-$  – which we know to be impossible. (I'm sorry, I haven't put this very elegantly, and who knows, I may have reversed an inequality sign somewhere, and it may all be nonsense.)

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No, it looks good -- I get it now.

Thanks.

quasi

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