Re: Limit of a sequence

Source: http://sci.tech−archive.net/Archive/sci.math/2008−04/msg02928.html

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- *Date*: Wed, 16 Apr 2008 00:58:01 EDT

> In article
> <15986032.1208228781859.JavaMail.jakarta@xxxxxxxxxxxxx
> forum.org>,
> TCL <tlim1@xxxxxxx> wrote:
>
> > For positive integers n, define
> >
> > s_n = sum_{k=1 to n} of (1/k)* (1−(1−1/n)^k).
> >
> > Prove that s_n −−> 1 as n−−>infty.
>
> n
> −−− n k 1
> > − (1 − (1 − 1/n)) −
> −−− k n
> k=1
>
> is a Riemann Sum for
>
> |\1 1 −x
> | − (1 − e) dx
> \| 0 x
>
> with x = k/n.

As noted by WWW, this is not quite a Riemann sum. To justify your steps, one needs to prove

Sum_{k=1 to n} (1/k)* (exp(−k/n) − (1−1/n)^k)

converge to 0 as n−−>infty. The fact that lim (1−1/n)

^n=e^{−1} may be helpful, but that does not prove that

the above limit is 0.

TCL

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