Re: Local Homeomorphisms

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- *From*: William Elliot <marsh@xxxxxxxxxxxxxxxxxx>
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On Thu, 17 Apr 2008, Jannick Asmus wrote:

> On 17.04.2008 09:26, William Elliot wrote:
>
> > > > Let f:X −> Y be a local homeomorphism.
> > > >
> > > > If X is connected, then is f a homeomorphism?
> > >
> > > No. Counterexample: C −> C\{0}, z −> exp(z);
> >
> > f:C −> C\0, z − e^z
>
> I meant z −> e^z.

Indeed, my typo.

> To provide a far simpler one: Z −> Z, x −> 0, Z equipped with the discrete topology.

Oh, I get it, local homeomorphisms don't have to be injective.

> I am so sorry for my bad English, I just literally translated the expression from my mother tongue. I meant a topological covering _map_.

Your English is good. Problem is translating technical expressions.
By your email address, you are German? I'm American.

> > > > If U,V are open and U homeomorphic f(U),

V homeomorphic f(V),
is not U \/ V homeomorphic to f(U) \/ f(V) =
f(U \/ V) ?

The simple example given above shows this is incorrect.

Sorry about the confusion. Yes − if you mean to take *one* of the counter examples above and patch two open subsets U and V such that the map on U u V is *not* injective.

Not even if U and V are overlapping?

Do not know what you are saying here. I suspect that you were not having the counter examples above in mind when you said this.

What if U /\ V is not empty? The simple counter example Z, fails.

I've a simple three point T0 space that's a counter example.
If I warp the line around S^1, ie
f:R −> S^1, x −> (sin x, cos x)

then I can find a counter example,
U = (−e, pi + e), V = (−pi − e, e)

for most small e. That's because the union isn't injective.

Well ok, thanks for clarifying that. In the application I'm considering, (see post "Spaces consisting of solely of split points" sci.math, April 14) the map in question is a bijection, a continuous bijection from a connected space. Does that change the results?

The first example, exp(z) isn't a bijection.
The second example, f:C\0 −> C\0, z −> z^k is a bijection?

I don't know enough about complex analysis to fathom this example.

However, I doubt that a connected space for which every point is a cut point is other than simply connected. Hopefully I won't have to include that hypothesis to get the conclusion that seems likely. That continuous bijection and perhaps connected will suffice.

The result I'm wishing for is
a bijective local homeomorphism is a homeomorphism.

Locally compact is another property of the domain space.
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