

# Local Homeomorphisms

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On Fri, 18 Apr 2008, Jannick Asmus wrote:

On 18.04.2008 12:13, William Elliot wrote:

Continuous  $f: X \rightarrow Y$  is a local homeomorphism when for all  $x$ , some open  $U$  nhood  $x$  with  $U$  homeomorphic  $f(U)$  and  $f(U)$  open.

Then clearly local homeomorphisms are open and when surjective are quotient maps.

Why is it that  $f(U)$  needs to be open?

This is just a question of definition, hence convention. I do not argue about definitions.

What goes wrong if  $f(U)$  isn't open?

Locally compact is another property of the domain space.

Apparently not needed.

Always depends on what you mean with "local homeomorphism". Some people use the wording "(local) homeomorphism on the image of  $f$  equipped with the trace topology". This makes it clearer in some cases. I think this is what you are talking about.

What's the trace topology?

## Local Homeomorphisms

Let  $f: X \rightarrow Y$  be a continuous bijection,  $X$  locally compact,  $Y$  Hausdorff.  
Is  $f$  a local homeomorphism? Well clearly for all  $x$ , some  
open  $U$  nhood  $x$  with  $U$  homeomorphic  $f(U)$ , but is  $f(U)$  open?

Certainly not: identity map  $(\mathbb{R}, \text{discrete topology}) \rightarrow (\mathbb{R}, \text{norm topology})$ .

Another counter example is  $f: [0,1) \rightarrow S^1$ .

It appears than in the notion of covering map, that the local  
homeomorphism has  $f(U)$  being open, not by the definition of  
local homeomorphism, but by the definition of covering map.

In other words  
covering map  $\rightarrow$  local homeomorphism with open local images.

Some compact  $K$  with  $x$  in  $\text{int } K$   
 $f: \text{int } K \rightarrow Y$  closed continuous bijection.  
 $\text{int } K$  homeomorphic  $f(\text{int } K)$

Why is  $f(\text{int } K)$  open, or is it?

HTH.

HTH ?

Convention – like "nhood". I do not want to argue about something like  
this. ;)

What's the definition of HTH?

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