

Re: Questioning the definitions of set and element.

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In article <oCISj.1209\$7z4.15@xxxxxxxxxxxxxxxx>, Mark <user@xxxxxxxx> wrote:

I don't see how a logical theory can be based on the undefined.

Quoted from "Introduction to Logic and to the Methodology of Deductive Sciences", by Alfred Tarski, translated by Olaf Helmer; Dover Publications, Inc., New York, unabridged republication of the 9th printing, 1961, of the 1946 second revised edition. ISBN 0-486-28462-X, pp 117 ff. What appears in the original in smallcaps font I place in quotation marks.

We shall now attempt an exposition of the fundamental principles which are to be applied in the construction of logic and mathematics. The detailed analysis and critical evaluation of these principles are tasks of a special discipline, called the "methodology of deductive sciences" or the "methodology of mathematics." For anyone who intends to study or advance some science it is undoubtedly important to be conscious of the method which is employed in the construction of that science; and we shall see that, in the case of mathematics, the knowledge of that method is of particular far-reaching importance, for lacking such knowledge it is impossible to comprehend the nature of mathematics.

The principles with which we shall get acquainted serve the purpose of securing for the knowledge acquired in logic and mathematics the highest possible degree of clarity and certainty. From this point of view a method of procedure would be ideal, if it permitted us to explain the meaning of every expression occurring in this science and to justify each of its assertions. It is easy to see that this ideal can never be realized. In fact, when one tries to explain the meaning of an expression, one uses, of necessity, other expressions; and in order to explain, in turn, the meaning of these expressions, without entering into a vicious circle, one has to resort to further expression again, and so on. We thus have the beginning of a process which can never be brought to an end, a process which,

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figuratively speaking, may be characterized as an "infinite regree" – a regressus in infinitum. The situation is quite analogous as far as the justification of the asserted statements of the science is concerned; for, in order to establish the validity of a statement, it is necessary to refer back to other statements, and (if no vicious circle is to occur) this leads again to an infinite regress.

By way of a compromise between that unattainable ideal and the realizable possibilities, certain principles concerning the construction of mathematical disciplines have emerged that may be described as follows.

When we set out to construct a given discipline, we distinguish, first of all, a certain small group of expressions of this discipline that seem to us to be immediately understandable; the expressions of this group we call "PRIMITIVE TERMS" or "UNDEFINED TERMS," and we employ them without explaining their meaning. At the same time, we adopt the principle: not to employ any of the other expressions of the discipline under consideration, unless its meaning has first been determined with the help of primitive terms and of such expressions of the discipline whose meanings have been explained previously. The sentence which determines the meaning of a term in this way is called a "DEFINITION", and the expressions themselves whose meanings have thereby been determined are accordingly known as "DEFINED TERMS."

We proceed similarly with respect to the asserted statements of the discipline under consideration. Some of these statements which to us have the appearance of evidence are chosen as the so-called "PRIMITIVE STATEMENTS" or "AXIOMS" (also often referred to as "postulates", but we shall not use the latter term in this technical meaning here); we accept them as true without in any way establishing their validity. On the other hand, we agree to accept any other statement as true only if we have succeeded in establishing its validity, and to use, while doing so, nothing but axioms, definitions, and such statements of the discipline the validity of which has been established previously. As is well known, statements established in this way are called "proved statements" or "theorems", and the process of establishing them is called a "proof". More generally, if within logic or mathematics we establish one statement on the basis of others, we refer to the process as a "derivation" or "deduction", and the statement established in this way is said to be "derived" or "deduced" from the other statement or to be their "consequence".

Contemporary mathematical logic is one of those disciplines which are constructed in accordance with the principles just stated[.]

[...]

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The method of constructing a discipline in strict accordance with the principles laid down above is known as the "deductive method"; and the disciplines constructed in this manner are called "deductive theories" [footnote omitted]. The view has become more and more common that the deductive method is the only essential feature by means of which the mathematical disciplines can be distinguished from all other sciences; not only is every mathematical discipline a deductive theory, but also, conversely, every deductive theory is a mathematical discipline (according to this view deductive logic is also to be counted among the mathematical disciplines). We will not enter here into a discussion of the reasons in favor of this view, but merely remark that it is possible to put forward ponderable arguments in its support.

Note in particular:

"we distinguish [...] a certain small group of expressions of this discipline[;] the expressions of this group we call "PRIMITIVE TERMS" or "UNDEFINED TERMS," and ->we employ them without explaining their meaning<-."

And

"Some of these statements [...] are chosen as the so-called "PRIMITIVE STATEMENTS" or "AXIOMS" [...]; ->we accept them as true without in any way establishing their validity.<-"

Primitive terms are "employed without explaining their meaning", and axioms are "accepted as true without in any way establishing their validity".

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"It's not denial. I'm just very selective about what I accept as reality."  
--- Calvin ("Calvin and Hobbes" by Bill Watterson)  
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