

Re: Geometry with right triangle..

Re: Geometry with right triangle..

Source: <http://sci.tech-archive.net/Archive/sci.math/2008-05/msg00180.html>

- *From:* mitch.nicolas.raemsch@xxxxxxxxxx
 - *Date:* Thu, 1 May 2008 14:44:05 -0700 (PDT)
-

On May 1, 10:33 am, bill <b92...@xxxxxxxxxx> wrote:

On Apr 30, 10:34 pm, "mina_world" <mina_wo...@xxxxxxxxxx> wrote:

Hello teacher~

The perimeter of a right triangle ABC is $2a$.

Find the scope of hypotenuse x .

Answer : $2\{\sqrt{2} - 1\}a \leq x < a$.

Ok, let's go...

hypotenuse : x
base : c
altitude : d

$$\begin{aligned}c + d + x &= 2a \\c^2 + d^2 &= x^2 \\c + d &> x\end{aligned}$$

Re: Geometry with right triangle..

(i) If $x \geq a$, then $c + d = 2a - x \leq a \leq x$.
contradiction.

(ii) By Cauchy–Schwarz inequality,
 $(c + d)^2 \leq (1^2 + 1^2)(c^2 + d^2)$
so, $(c + d)^2 \leq 2 \cdot (x^2)$
so, $(2a - x)^2 \leq 2 \cdot (x^2)$
so, $2 \cdot (x^2) - (2a - x)^2 \geq 0$
so, $2 \cdot (x^2) - [4a^2 - 4ax + x^2] \geq 0$
so, $x^2 + 4ax - 4a^2 \geq 0$
so, $x \leq 2a\{-1 - \sqrt{2}\}$, $x \geq 2a(-1 + \sqrt{2})$

so, $2\{\sqrt{2} - 1\}a \leq x < a$.

x is minimum when $c+d$ is maximum.
Guessing that $c+d$ is maximum when
 $c = d$; we surmise that

$$\begin{aligned} 2c^2 &= x^2 \quad \text{Then} \\ \sqrt{2}c &= x = \sqrt{2}d \\ c &= x/\sqrt{2} = d \\ 2a &= x/\sqrt{2} + x/\sqrt{2} + x \\ &= \sqrt{2}x + x \\ 2a &= x(1 + \sqrt{2}) \\ 2a/[1 + \sqrt{2}] &\leq x \end{aligned}$$

Bill J– Hide quoted text –

– Show