Re: opponents of taylor and l'hospital ?

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On May 4, 11:29 pm, lwal...@xxxxxxxxx wrote:

> In this thread below, Dr. Gerry Myerson does give a valid reason
>
> Suppose a high school senior taking the AP Calculus exam
> this month sees the following problem:
>
> lim (x−>0) (sin x)/x =
> A) 0
> B) 1
> C) −1
> D) does not exist
>
> If I were this student, I'd simply use L'Hopital's rule and find
> the anwer to be (cos x)/1 = cos 0 = 1 in seconds. Why
> should I care that the limit is used in the proof that
> d/dx (sin x) = cos x? The question didn't ask to prove that
> d/dx (sin x) = cos x −− it asked to find the limit, and the
> fastest way to find the limit is L'Hopital's rule.

There are other ways to show that d/dx(sin x) = cos x, without computing the limit of (sin x)/x first. If you start with defining complex exponentials e^z as the series \sum_{n=0}^{|infty} z^n/n!, and then define e^{ix} = C(x) + iS(x), it immediately follows that d/dx(S) = C, by observation of series representations of C(x) and S(x). The only remaining part is to show that both C and S match the regular cos and sin we learned in high school. Once we have this in place, we can certainly use the L'Hospital rule to show that the limit is 1.

BUT practically all Calculus textbooks derive the limit of sin(x)/x first, using pure geometric means, and *then* use it to show that d/dx(sin) = cos x. With this in mind, using L'Hospital rule to argue that the limit is 1 *is* a circular reasoning. That's why you should care how d/dx (sin x) = cos x was obtained in the first place. (Of course there is a reason why Calculus does not do it 'series' way: you have to have all the series related staff in place, so that's normally done in analysis. Baby Rudin does it this way.)

> In fact, to this day I don't remember how to prove the differentiation rules for sin x and e^x −− yet I still recall lim (x−>0) (six x)/x = lim (x−>0) (e^x−1)/x = 1, thanks to L'Hopital's rule. Except for polynomial functions, I cannot differentiate any function from first principles only.

Well, there are different ways to define e^x. Sometime you start with ln x first, as the integral of 1/x, then define e^x as inverse function of ln x, then all the properties including d/dx e^x = e^x follow. Of course in case of series representation, the fact that d/dz e^z = e^z immediately follows, by observation.

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