# Re: Percentage of Prime Numbers

***Source:*** http://sci.tech−archive.net/Archive/sci.math/2008−05/msg01711.html

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- *From*: Mensanator <mensanator@xxxxxxx>
- *Date*: Sun, 11 May 2008 21:58:17 −0700 (PDT)

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On May 11, 8:46 pm, Narcoleptic Insomniac
<i_have_narcoleptic_insom...@xxxxxxxxx> wrote:

> On May 11, 2008 2:37 PM CT, orangata...@xxxxxxxxxxxxxx wrote:
>
>> On 11 May, 13:12, Narcoleptic Insomniac
>> <i_have_narcoleptic_insom...@xxxxxxxxx> wrote:
>>
>>> On May 11, 2008 6:36 AM CT, ad...@xxxxxxxxxxxx
>>> wrote:
>>>
>>>> Hi,
>>>>
>>>> I just checked my 10−€−Banknote and found
>>>> out the
>>>> serial number is a prime number. I just asked
>>>> the
>>>> European Central Bank how many
>>>> Banknotes circulate
>>>> and want now to find out the chance that a
>>>> randomly
>>>> picked banknote has a prime serial number.
>>>> So − is
>>>> there any possibility (without crashing my
>>>> PC) to
>>>> find out how many prime numbers are in a
>>>> given area
>>>> of numbers?

Thanks a lot!

Yes, one way would be to use approximations of the prime counting function (denoted pi(x)). The Prime Number Theorem tells us that this function is asymptotic to x / ln(x), or in other words

pi(x) ~ x / ln(x).

So, suppose that you wanted to know roughly how many primes were between A and B (given B > A). Then, by the PNT, we know that

pi(B) − pi(A) ~ B / ln(B) − A / ln(A)

= B ln(A) − A ln(B) / [ln(B) ln(A)]

= ln(A^B) − ln(B^A) / [ln(B) ln(A)]

= ln(A^B / B^A) / [ln(B) ln(A)]

..is roughly the number of primes between A and B.

Of course, you can divide this by (B − A) to get an approximate density.

Regards,
Kyle Czarnecki

> > that is cool. any idea why?

> Yes, I've always found number theory to be one of the most beautiful branches of pure mathematics. As for "any idea why," I'm not sure exactly what you're asking.
>
> Out of curiosity, I've calculated a similar example of the above method using U.S. currency. The serial numbers on the U.S. one dollar bill (2006 series) have the form
>
> A 12345678 B
>
> ..that is, eight digits between two letters. I am unsure of how these digits are assigned, but let us first assume that they range from 00000000 to 99999999. It follows that there are 10^9 possible serial numbers (neglecting alphabetic characters).

It would appear to be more than that. The first letter is the code for the Federal Reserve Bank. The second is the run number. I think this means a particular prime can appear 26 times for each of the 12 FRBs. And I got the impression from various web sites that each sheet of paper contains 32 bills with the same number. They claim 32*26 or 832 bills with the same number (not counting the FRB).

> Moreover, the PNT states that
>
> pi(10^9) ~ 10^9 / ln(10^9) ~= 48254942
>
> ..which implies that there are approximately 48254942 primes between 0 and 10^9. Thus, the density of primes in this interval is
>
> pi(10^9) / 10^9 ~ 1 / ln(10^9) ~= 0.048254942
>
> ..and hence about 4.8255% of U.S. one dollar bills (in this case) will have a prime serial number.
>
> It should be mentioned that there exist better approximations to the prime counting function; namely,
>
> pi(x) ~ li(x)

..where li(x) is the logarithmic integral of x.  (I think that someone this thread already pointed out this fact.) If we use this approximation, then

pi(10^9) ~ li(10^9) ~= 50849234.957

..which implies...

pi(10^9) / 10^9 ~= 0.050849234957

..and so, using this approximation, we see that (in this case) about 5.0849% of U.S. one dollar bills have a prime serial number.

Actually, Mathematica says that pi(10^9) = 50847534, and so the density is

pi(10^9) / 10^9 = 0.050847534

..which implies about 5.0848% of U.S. one dollar bills (in this case) will have a prime serial number.

Note how much closer the li(x) approximation is.  I don't know about you, but I find it a bit suprising that approximately 1 in 20 $1 bills has a prime serial! *_*

Again, purely out of curiosity, let us take a look at a similar problem.  Suppose that we have a U.S. dollar bill whose serial begins with the digit 9.

What are the chances that such a dollar bill contains a prime serial number?  Well, this amounts to counting the primes between 90000000 and 99999999.

Using the pi(x) ~ x / ln(x) approximation yeilds

pi(99999999) − pi(90000000) ~ 99999999 / ln(99999999) − 90000000 / ln(90000000) ~= 514761.975776

..and since there are 10^7 integers in this interval, the density is...

[pi(99999999) − pi(90000000)] / 10^7 ~= 0.0514761975776.

Hence, approximately 5.1476% of such U.S. one dollar bills will have a prime serial number.

Using the pi(x) ~ li(x) approximation instead yeilds

pi(99999999) − pi(90000000) ~ li(99999999) − li(90000000) ~= 544399.097718

> ..and so the density is about...
>
> [pi(99999999) − pi(90000000)] / 10^7 ~= 0.0544399097718
>
> ..whicih implies approximately 5.4440% of such U.S. one dollar bills will have a prime serial number.
>
> However, Mathematica says that
>
> pi(99999999) − pi(90000000) = 544501
>
> ..so the density is...
>
> [pi(99999999) − pi(90000000)] / 10^7 ~= 0.0544501
>
> ..and hence about 5.4450% of such U.S. one dollar bills has a prime serial.
>
> It is interesting to note that, even though we've chosen the first digit to be 9, we still see that about 1 in 20 U.S. dollar bills will have a prime serial.

I just happened to have a wad of 28 one dollar bills and found:

04284315 x
20948480 x
18823776 x
25035494 x
20298387 x
41328940 x
09083777 x
77950606 x
74764206 x
19692481 prime
79574678 x
90007632 x
90007631 x
77409753 x
93215626 x
13238183 x
40463897 x
30162626 x
53558093 x
40728940 x
27475180 x
92293006 x
49975428 x
80104290 x

33550351 prime
82784801 x
52471303 x
84361182 x

No big surprise there, but here's a surprise.

The FRB distribution was

Federal Reserve Banks:
2 **
3 **
4 ****
5 *
6 ******
9 *
11 *****
12 *******

which, being from the Chicago area, it is quite unusal to have 28 bills without a single instance of the local Chicago FRB (7).

Regards,
Kyle Czarnecki− Hide quoted text −

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