Re: continuous iteration

Source: http://sci.tech−archive.net/Archive/sci.math/2008−05/msg03759.html

- *From*: galathaea <galathaea@xxxxxxxxx>
- *Date*: Sun, 25 May 2008 13:14:38 −0700 (PDT)

On May 25, 11:59 am, Tonico <Tonic...@xxxxxxxxx> wrote:

> On May 25, 8:44 pm, galathaea <galath...@xxxxxxxxx> wrote:
>
>> On May 22, 11:57 pm, Tonico <Tonic...@xxxxxxxxx> wrote:
>>
>>> On May 23, 2:48 am, galathaea <galath...@xxxxxxxxx>
>>> wrote:
>>>
>>>> sometimes
>>>> as in the case of gamma
>>>> you need to add conditions for uniqueness
>>>> (like log convexity does with
>>>> bohr−mollerup)
>>>> (and it seems like in your last message
>>>>
>>>>> Tetration seems to be a
>>>>> pretty edgy subject,
>>>>> and since it apparently uses
>>>>> the well known
>>>>> Gamma function, poor
>>>>> tommy thinks (so to speak)
>>>>> that if someone doesn't
>>>>> know about tetration
>>>>> then he also doesn't know
>>>>> about the Gamma
>>>>> function...:)
>>>>
>>>> you may not be noticing gamma itself is a
>>>> continuous iterate)

there has never been anything "edgy" about
tetration

it has a long history

****

I don't think so: I think it's hugely edgy, but it doesn't really
matter: you may as well not consider it so.
About gamma being continuous iterate: do you mean that it
fulfills

G(s+1) = s*G(s), or what? I never heard it called that way,
but that
jsut could be my bad. I hear id called "the factorial function",
we
even proved in an advanced calculus course that G is the
unique real
function which fulfills:
1) G a log−convex function;
2) G(1) = 1
3) G(s+1) = sG(s)

the factorial of n can be seen as
"multiply the n numbers from 1 to n together"

1.2.3...n

which can be written in the rising pochhammer

(1)
n

now
much as i demonstrated for changing
"sum the n numbers f(.)..."
there is a way to make sense of "continuous products"

again
much as with sums
the idea is to extend the iteration infinitely
with a shifted part that normally cancels
but some extra care to avoid infinities is necessary

write

(1) (1) (1+n)
x+n n x
(1) = −−−−−− = −−−−−−−−−−
n (1+x) (1+x)
n n

^^^^
ugh− (1)_x not n
this probably caused some confusion

now
the leading term of

(1+n)
x

x
is n

so the limit as n−>oo of the above
is the same limit as

(1) x
n n
lim −−−−−−−−
n−>oo (1+x)
n

which is defined for all x
and is one way euler defined

__
| '
| (1 + x)

..

so
now i've shown you ways of making continuous
iterative processes for sums and products

do you see how iterative exponentiation
can also be so generalised?

several different ways have been proposed
but there is one approach that follows this same idea

can you guess it now?

****

I sincerely thank you for trying to make some sense for me out of all the above, but I still can't see it: you began by saying that

n! = (1)~ n −− meaning (1) is kind of power to the left of n −−.

Then you write (1)~(x + n), which I can udnerstand as being (x + n)! generalized, most like the infinite product in (a+b)^1/2 as series,

> trying to generalize Newton.
>
> But then you write (1+x)~ n, and I just can't understand what this mean: factorial from 1+x to n or what..??

i'm sorry for the mistake noted above
and hope the correction clears a lot up

basically
the problem is something defined as
"multiplying up from 1 to some n"
wants to be generalised to a continuous number of times

the task then is to build a path of equalities
that takes a given expression that makes sense for discrete values
and transform it into something that makes sense for all real values
(or all complex values − which comes with the analytic theory of real functions)
(compactified)

> Not that iteration or whatever has anything to do with the solution of that darn old hoax, but it would nevertheless be nice to understand something new in maths.

it kind of does
though

the sum is a discrete operation
but it has dependent variable n
so the differential operator cannot act through the sum

ie.

```
n n
−−− −−−
d \ \ dn
−− / n =/= / −− (=1)
dn −−− −−− dn
j=1 j=1
```

but
there is an obvious way to solve this impasse

the inner expression does not depend on j
so as the very first post of the trick points out
there is a basic sum identity that maps this to multiplication

i.e.

```
n
---
\ 2
/ n = n n = n
---
j=1
```

which is an expression that can be generalised to continuous n
and therefore differentiated

a lot of historians
when they look back on euler doing transformation like this
have instilled a fear in a lot of mathematicians
that (hand wring) it's not really "valid" to do these kinds of things

but this can all be rigorously founded
and all of the basic mappings of properties that carry over at each stage
have well established theories

definitions that apply to Z can often be lifted in "natural" ways
to R or C or many other (rings, fields, ..)

this can all be algebraicised

-=-
galathaea: prankster, fablist, magician, liar
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