

Re: -- Limit of ratio of consecutive primes = 1 ?

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- *From:* bill <b92057@xxxxxxxxxx>
 - *Date:* Sun, 25 May 2008 17:44:10 -0700 (PDT)
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On May 24, 5:31 am, David C. Ullrich <dullr...@xxxxxxxxxx> wrote:

On Fri, 23 May 2008 13:26:39 -0700 (PDT), bill <b92...@xxxxxxxxxx> wrote:

On May 22, 5:12 am, David C. Ullrich <dullr...@xxxxxxxxxx> wrote:

On Wed, 21 May 2008 11:48:23 -0700 (PDT), bill <b92...@xxxxxxxxxx> wrote:

On May 21, 9:34 am, David W. Cantrell <DWCant...@xxxxxxxxxx> wrote:

Ray Johnstone
<r...@xxxxxxxxxx>
wrote:

On 07 May
2008
23:32:48
GMT,
David W.
Cantrell
<DWCant...@xxxxxxxxxx>
wrote:

Let
 $p(n)$
denote
the
 n th prime.

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It
seems
that,
as
 $n \rightarrow \infty$,
 $\lim_{n \rightarrow \infty} \frac{p(n+1)}{p(n)}$
=
1.
Is
that
correct?
If
so,
how
can
it
be
proven?

Does
it
perhaps
depend
on
some
result
(unfamiliar
to
me)
concerning
how
big,
for
given
 n ,
the
gap
between
 $p(n)$
and
 $p(n+1)$

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can
be?

So is there a
limit? There
seems to be
no answer
in this long
thread.
If there is
one it seems
to be
between 1
and 2.

The limit is 1. That was
proven early in the thread.

David

The first order limit is $\Rightarrow 1$.

What in the world does that mean?

The limit of $P(n+1)/P(n)$ is 1.
Another way to say that would be to say that
 $P(n+1)/P(n) \rightarrow 1$. But the limit is 1,
it does not "tend to 1". And nobody but
you knows what a "first order limit" is.

Since $P(n+1)$ cannot be less than $P(n)$;
the limit
must be $\Rightarrow 1$. . .

.
Twin primes quickly push the limit to 1
exactly.

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Huh? That shows that limit is 1 assuming two things: (i) there are infinitely many twin primes, (ii) the limit exists. But nobody knows whether (i) is true, and (ii) is not something you're allowed to assume here, it needs to be proved.

I understand now why you did not bother to explain this to me earlier..

I would have to be able think like a mathematician to make any sense out of the above

The reason I didn't say anything about those points earlier is that they were not relevant to anything you'd said. The same applies to almost all of the math I know.

All of this has been based on a certain theorem known as thePrimeNumber Theorem, which says that a certain other limit is 1. That is a very deep theorem. But if you simply assume that the limit in question exists the PNT becomes quite simple.

Way over my head!

Not every sequence has a limit. An example of a sequence that does not have a limit is given by one of your other erroneous comments about the limit of $\sin(n)$; that sequence does not have a limit. A simpler example is this:

1, -1, 1, -1, 1, -1, ...

That sequence does not have a limit (very informally, the limit can't be 1 because there are infinitely many -1's in the sequence, the limit can't be -1 for similar reasons, and the limit certainly can't be anything other than 1 or -1.)

If $\pi(n)$ is the number of primes less than or equal to n thePrimeNumber Theorem says that the sequence defined by

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$\pi(n) \cdot \ln(n) / n$

has limit 1. That theorem is very difficult. But if we assume that that sequence _has_ a limit it's "easy" to prove that the limit must be 1; the very hard part of the proof is showing that the sequence _has_ a limit.

I have forgotten what this has to do with the limit $[P(n+1) / P(n)]$.

$P(n+1)$ is asymptotic to $(n+1) \cdot \ln(n+1)$ and
 $P(n)$ is asymptotic to $n \cdot \ln(n)$
For significantly large 'n', these two curves are virtually identical. This might be useful information.

Bill J

David C. Ullrich

David C. Ullrich