Re: Linear algebra with eigenvalue AB.

Source: http://sci.tech−archive.net/Archive/sci.math/2008−06/msg00817.html

- *From*: Bill Dubuque <wgd@xxxxxxxxxxxxxxxxxxxx>
- *Date*: 09 Jun 2008 18:35:21 −0400

Tonico <Tonicopm@xxxxxxxxx> wrote:

> Bill Dubuque <wgd@xxxxxxxxxxxxxxxxxxxx> wrote:
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>> Tonico <Tonicopm@xxxxxxxxx> wrote:
>>
>>> Fair enough: something like is what I suposed he was trying
>>> to do, and
>>> again the question is: what for? Mina's question was very
>>> specific and
>>> simple, and he (together with with the help of others) already
>>> covered
>>> all the possible cases in her problem: det A different from
>>> zero, and
>>> equal to zero. Why then to make more messy simple stuff?
>>
>> Not true. The only other proof mentioned here for the case det(A)=0 was a reference by Robert Israel to a prior thread, where Robin Chapman says that it can be proved by a topological argument (but gave no details). Such a proof is far more complex than the trivial one−line algebraic proof I presented above. It seems you have not followed my suggestion to read my prior posts where I gave many more examples (some simpler) illustrating such universal techniques. This is a very simple yet very powerful technique in algebra that every mathematician should learn to master. Often times one will see analysts struggle with obfuscated topological density arguments when all that is required is a simple algebraic argument exploiting the universality of polynomials. This is something one should learn in a first course in abstract algebra but, alas, it seems many people don't. The fact that you are continuing to struggle to comprehend something that is utterly trivial only serves as further evidence reinforcing my oft−stated claim here about how widely misunderstood is the simple algebraic notion of a polynomial. I've probably written over 50 posts discussing many variations on this theme, so please take the time to read a few of them if you honestly want to learn something about it. Here is the link again. FOLLOW ITS LINKS, ETC, ETC to traverse the entire tree of prior posts
>> http://google.com/groups?threadm=y8zsl3e3nuy.fsf%40nestle.csail.mit.edu

> Thanx for the links. it's a little messy to follow those (I already reached once 1999 and other time 2000...), so: do you have some site where these techniques are written? For example, I tried to read about the |Adj A| = |A|^(n−1) but I only found links to other posts with links to other post with links to...*pant*. If all this is more or less concentrated in some place, book or articles, it'd be great.

That example I never elaborated on. It is probably too close to the example in this thread to be of help pedagogically. Let's consider a simpler example of defining derivatives of polynomials algebraically. Given a poly f(x) in R[x] define the derivative f'(x) as follows:

f'(x) = g(x,x), where g(x,y) := (f(x)−f(y))/(x−y) in R[x,y]

Note that the existence and uniqueness of this derivative follows from the Factor Theorem, i.e. x−y divides f(x)−f(y) in R[x,y], and from the cancellation law, i.e. (x−y) g = (x−y) h in R[x,y] implies g = h. It is clear this agrees with the usual definition since it is linear and it takes the same value on monomials x^n.

Now we algebraically prove the product rule rule for derivatives:

f(x)g(x) − f(y)g(y) = [f(x)−f(y)] g(x) + f(y) [g(x)−g(y)]

−> (fg)' = f' g + f g'

PROOF: divide the first equation by x−y, then evaluate at y = x, i.e. the difference quotient from the product rule for differences

The cancellation of the factor x−y in the above inferences is precisely analogous to the cancellation of det(A) in the example sparking this thread. It is valid only because f, g are formal polynomials (vs. arbitrary (real) functions).

Note how the above proofs avoid any use of topological concepts (limits, continuity, etc), Instead such notions are replaced by the purely algebraic notions of evaluation, and cancellation. One may view that cancellation law as one of the simplest examples of a uniqueness theorem. As I've stressed here on many occasions, uniqueness theorems provide powerful tools for proving equalities. This example is discussed a little bit further in my prior post
http://google.com/groups?selm=y8z1ya4tmcy.fsf%40nestle.ai.mit.edu

−−Bill Dubuque
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