Re: Matrix Algebra question

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- *From*: Narcoleptic Insomniac <i_have_narcoleptic_insomnia@xxxxxxxxx>

- *Date*: Sat, 14 Jun 2008 02:08:25 EDT

On Jun 13, 2008 10:02 PM CT, TCL wrote:

> Let L_2 be the 2x2 lower triangular matrix whose
> nonzero off diagonal entry is 2, i.e. a11=1, a12=0,
> a21=2, a22=1. Let U_2 be its transpose.
> I am looking for an easy proof of the following fact:
>
> The group (with matrix multiplication) generated by
> {L_2, U_2} is the set of matrices A with a11, a22 odd,
> and a21, a12 even, and det(A)=1.
>
> A direct proof seems to be not easy.

Here are some facts that may help (I will denote L_2 and
U_2 as just L and U, respectively).

First, we can show that L (and hence U) has a unit
determinant. For if L = [a_11, 0; 2, a_22], then L^{−2}
= [a_22, 0; −2, a_11]. Thus, det(L) = a_11 a_22 =
det(L^{−1}), so

1 = det(I) = det(L L^{−1}) = det(L) det(L^{−1})

...which implies...

det(L) = det(L^{−1}) = +− 1.

Furthermore, it is not difficult to diagonalize L by
finding the eigenvalues and eigenvectors −− specifically

lambda_1 = a_11 with v_1 = [d, 2], and

lambda_2 = a_22 with v_2 = [0, 1]

...where d = a_11 − a_22. It follows from this that

L = P J P^{−1}

...where J = [a_11, 0; 0, a_22], P = [d, 0; 2, 1],
P^{−1} = [1/d, 0; −2/d, 1]. Note that this implies

L^n = P J^n P^{−1}

...for all positive integers n. Moreover, since U = L^t
we see that

U = L^t = (P J P^{−1})^t = P^t J (P^{−1})^t.

Again, note that this implies that

U^n = P^t J^n (P^{−1})^t

...for all positive integers n.

Maybe we can use these general facts of L^n and U^n to
prove your proposition about G = < L, U >.

Regards,
Kyle Czarnecki
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