Re: Matrix Algebra question

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On Fri, 13 Jun 2008 20:02:57 −0700 (PDT), TCL <tlim1@xxxxxxx> wrote:

> Let L_2 be the 2x2 lower triangular matrix whose nonzero off diagonal
> entry is 2, i.e. a11=1, a12=0,
> a21=2, a22=1. Let U_2 be its transpose.
> I am looking for an easy proof of the following fact:
>
> The group (with matrix multiplication) generated by {L_2, U_2} is the
> set of matrices A with a11, a22 odd, and a21, a12 even, and det(A)=1.
>
> A direct proof seems to be not easy.

There's a simple "direct" proof in Nehari "Conformal Mapping":
Say the group generated by those two matrices is G and the
group of all matrices such that a11 is odd[etc] is H.

It's trivial to check that G is a subgroup of H. For the
other direction: Define chi([[a,b],[c,d]]) = |a| + |c|.
Say A is in H, and let S = {BA : B in G}. Say C is
an element of S that minimizes chi. You can show
by contradiction that C_{2,1} = 0, and then it
follows that C is a power of U_2.

Or something like that...

> −TCL

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