

Re: Sharply 5-Transitive: M12-

Source: <http://sci.tech-archive.net/Archive/sci.math/2008-06/msg02444.html>

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 - *Date:* Tue, 24 Jun 2008 15:22:48 EDT
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PaulHjelmstad wrote:

The order of M12 is 95,040, which is $132 * 720$

Since M12 is the automorphism group of $S(5,6,12)$, and is sharply 5-transitive, and maps blocks to

blocks,

would there be 720 operations to go from say,

hexad(A)

to hexad(B), (and B to C, etc) such that

$x_1, x_2, x_3, x_4, x_5 \rightarrow y_1, y_2, y_3, y_4, y_5$; and every

combination

$x_1, x_2, x_3, x_4, x_5 \rightarrow$ (y's scrambled 5!) which

makes 120,

and then times a factor of 6, because there are 6

pentads in each hexad, such that $C_{6,1} = C_{6,5}$ pentads, to get every 720 g's going between two hexads? I sense it is more complex than this. I also am assuming all hexads are treated equally, which is probably wrong, such that

there are 132 cycles between hexads, going

A,B,C,...

A,C,E... (and B,D,F.), A,D,G etc

Or does sharply transitive mean there is only one

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(not 120) going between g1 through g5 -> h1 through h5?

Rotman seems to be beyond my scope, even though I understand some of the theorems and proofs for this

PGH

"Sharply transitive" means here that for each pair of pentads P1, P2 there is exactly one permutation in M12 that maps P1 onto P2. The pentads are 5-element subsets with no order relations; they are just subsets. For a specified M12 subgroup of S12 this means that once you have a permutation

$(x_1, x_2, x_3, x_4, x_5, \dots) \rightarrow (y_1, y_2, y_3, y_4, y_5, \dots)$

in which the entire x-subset is mapped onto the entire y-subset, you cannot determine which x_i is mapped onto which y_j .

One would expect that there exist 120 subgroups of S12 isomorphic to M12 and all of them mutually conjugate.

BTW, the Mathieu group M24 is transitive but not sharply transitive on pentads within a set of 24 elements. One would expect that within M24 indeed 120 permutations would map a given pentad onto another given pentad. I do not know by heart if this is true.

See ...

http://en.wikipedia.org/wiki/Transitive_group_action#Types_of_actions

http://en.wikipedia.org/wiki/Mathieu_group

Happy explorations: Johan E. Mebius

Thanks. That is what I thought, they are just sets. It would be too easy to merely permute the order. By mutually conjugate I take it that all 120 subgroups are in the same conjugacy class?

It would be fun to get a literal mapping of hexads, to which hexads, but I guess then I would need a full listing of all 95,040 g's in M12! I will continue to struggle with Rotman, and if 132 hexads are

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treated similarly, I guess 720 g's would explain things.

PGH

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