# Re: Sharply 5−Transitive: M12

***Source:*** http://sci.tech−archive.net/Archive/sci.math/2008−06/msg02526.html

- *From*: PaulHjelmstad <phjelmstad@xxxxxxx>
- *Date*: Wed, 25 Jun 2008 11:20:22 EDT

On 24 Jun, 18:42, PaulHjelmstad <phjelms...@xxxxxxx>
wrote:

> The order of M12 is 95,040, which is 132 * 720
>
> Since M12 is the automorphism group of S(5,6,12),
> and is sharply 5−transitive, and maps blocks to

blocks,

> would there be 720 operations to go from say,

hexad(A)

> to hexad(B), (and B to C, etc) such that
>
> x1, x2, x3, x4, x5 −> y1, y2, y3, y4, y5; and every

combination

> x1, x2, x3, x4, x5 −> (y's scrambled 5!) which

makes 120,

> and then times a factor of 6, because there are 6

pentads in each hexad, such that C6,1 = C6,5 pentads,
to get every 720 g's going between two hexads? I
sense it is more complex than this. I also am
assuming all hexads are treated equally, which is
probably wrong, such that

> there are 132 cycles between hexads, going

A,B,C,...

> A,C,E... (and B,D,F..), A,D,G etc

Or does sharply transitive mean there is only one

(not 120) going between g1 through g5 −> h1 through
h5?

Rotman seems to be beyond my scope, even though I

understand some of the theorems and proofs for this

PGH

I have found the answers you have received to this
confusing!

Saying G acts sharply 5−transitive on a set X means
that for any
distinct x1,x2,x3,x4,x5 in X and any distinct
y1,y2,y3,y4,y5 in X,
there exists exactly one permutation g that maps
xi−>yi for all i. So
the order matters here, and it means that there are
exactly 120
permutation in G that map the set {x1,x2,x3,x4,x5} to
{y1,y2,y3,y4,y5}.

So M12 sharply 5−transitive on 12 points means that
its order is
12*11*10*9*8 = 95040.

Saying a group is 5−transitive means that there is at
least
permutation with the above property − "sharply" means
exactly 1.

M24 on 24 points is 5−transitive but not sharply so.
In fact there are
48 permutations mapping any ordered pentad of
distinct points to any
other, so |M24| = 24*23*22*21*20*48 = 244823040.

Derek Holt.

Thanks Dr. Holt for your answer. I've been going over
my earlier questions on M24 (Steiner Systems), finally things are falling into place. So order does matter! I
have been studying the construction of M12 from the
inner and outer automorphisms of S6, Steven
Cullinane's Web pages have been fascinating (Finite

Geometry, Diamond Theorem). Also I should say JEMebius's
pages and applets are also pretty amazing.

PGH
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