tetration and logaritms

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we all know the taylor series for log(1+x) which holds for all real x between −1 and 1.

this series starts with 0 + x − x^2/2 + ...

and from looking at the first terms of this taylor series we see that we can get series reversion which of course gives exp(x) − 1.

fractional iterations can be done for exp(x)−1 since its a taylor series with f(0) = 0. (and its unique)

and thus also for log(1+x) , either directly since log(1) = 0 or from the series reversion of the fractional iterations of exp(x)−1.

so we can compute f(f(1+x)) = log(1+x) (*)
(for x between −1 and 1)

set v = x+1 for v between 0 and 2
f(f(v)) = log(v)

now we can thus compute the half−iterate of the log function.

(*) beware f(f(x+1)) is a taylor series in x but you need to convert to a taylor series in x+1 !!
use the substitution x = w−1 and the binomial theorem to compute it.

thus for v between 0 and 2 we can compute f(v) and
f(v) is the half iterate of log(v).

to compute the half iterate of e^v , simply use
exp(f(v)).

thus semi−exp(1) = exp(f(1))

despite f(2.71828) is undefined since e = 2.71828..
is larger than 2 we can now also compute semi−exp(e)

semi−exp(e) = exp(exp(f(1)).

all of this of course shows tetration does exist ;
unique continu iteration of exp(x) !!

tetration is by this construction smooth ...

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analytic too ?

natural question ; what about complex iterations ?

if a function is smooth but complicated

if analytic , we can apply analytic continuation.

if not analytic , how do we do a continuation ?

(i assume doing the analytic continuation of its inverse function , and that this works for tetration but im not sure , as for the general case if this does not work i dont know)

but note that i dont require tetration to be defined for complex numbers.

why not ?

an example : exp(x) is the continu iteration of e*x.

an iteration of a function is not suppose to have period.

(think about it !)

e.g. sine seems like an iteration of sqrt(1−x^2)

but look at the zero's of sin(q) = 1/5.

the zero's are not periodic , thus the iterations at the values 1/5 are not always the same iteration !!

thus sine does not represent a unique continue iteration.

exp(x) is on R since exp(x) is monic.

however exp(x) has a period too ; 2pi i !

thus in the complex sence exp(x) is not a unique continue iteration !!

THUS any function that has a period is not a unique continue iteration.

(or that function + some complex constant as demonstrated by sine)

i dont think any smooth function exists that is a unique continue iteration (in the complex sense) apart from a*z + b.

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this makes tetration a computable function , however the goal of a series expansion is still not reached.

since my solution is unique maybe we should drop the restriction of analytic on R+ to smooth (C oo).

(however it might be analytic on R+ afterall)

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pentation still seems out of reach ...

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regards

tommy1729

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