Re: Integer lattice

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- *From*: sanchopancho80@xxxxxx
- *Date*: Wed, 20 Aug 2008 04:09:07 −0700 (PDT)

On 20 Aug., 02:28, Mariano Suárez−Alvarez
<mariano.suarezalva...@xxxxxxxxx> wrote:

> On Aug 19, 6:52 pm, sanchopanch...@xxxxxx wrote:
>
>> Hello,
>>
>> perhaps it is a little boring to bring up this problem once again but
>> I have some problems with it, yet.
>>
>> Problem: Let f:Z^2−−>[0,oo) be a function such that f(n,m)=(f(n+1,m)
>> +f(n−1,m)+f(n,m+1)+f(n,m−1))/4 then f is constant.
>>
>> I have seen the following solution in a book with the
>> title...well...something with "newman" in the title, but however, here
>> is the solution stated there:
>> All such functions f form a locally compact convex cone. It suffices
>> to show that there is only one extreme point. Let T be the translation
>> (1,0) and S be the translation (0,1) one has
>> f=1/4*fT+1/4*fT^{−1}+1/4*fS+1/4*fS^{−1}
>> For an extreme point holds fT=xf and fs=yf and one gets x=y=1.
>>
>> I don't understand this solution completely. Why is it sufficient to
>> show that there is only one extreme point? In my opinion every cone
>> has only one extreme point...?
>
> Suppose you consider only bounded solutions to your

functional equation, and let S be the set of
solutions in L^infty(Z x Z). It is easy to see that
S is weak−star closed and that the intersection Q
of S and the unit ball B in L^infty(Z x Z) is a
weak−star compact convex set K (using the
Banach−Alaoglu theorem to get the weak−star
compacity of B)

Therefore, by the Krein−Milman theorem, C is the closed
(in the weak star topology) convex hull of its set of extremal
points. If you show that there is only one extremal point p, then,
you'll obviously have that C = { p }.

You seem to want, though, this for not−necessarily
bounded solutions, so you'll aparently need some
variant of Krein−Milman for non−compact convex sets.
I do not know of any such variant...

−− m

Thank you. Does anybody know such a variant?
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