Re: Rudin and Dedekind cuts

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On Wed, 20 Aug 2008 06:04:23 EDT, Mathman1271
<mathman1271@xxxxxxxxxxx> wrote:

> But surely there is a fundamental difference. To define rationals we can start with the Peano axioms for
> integers and then define rationals as ordered pairs of integers. The idea of a 'number' is very clear, there is no ambiguity.

No, we define rationals as certain _equivalence classes_ of ordered
pairs of integers. For example, 1/2 is the set of all (n,m) such that
n,m are integers, m is nonzero, and n = 2m.

I don't see why defining a real number to be a set of rationals with
certain properties bothers you more than defining a rational to
be a set of ordered pairs of integers with certain properties.

> With Dedekind cuts, there IS ambiguity.

What ambiguity are you referring to?

Hmm. There _is_ an "ambiguity" in what rational numbers
are. We start with the rationals. Then the reals are defined
to be certain sets of rationals. But the rationals are supposed
to be a subset of the reals...

Look back at your original post. We note that R contains a
subfield isomorphic to Q. If you want to get everything
perfectly straight you should think of two different
models of the rationals; two different isomorphic fields.
The orginal rational 1/2 is not the same thing as the
real number 1/2. Say 1/2_Q is the original one and
1/2_R is the new one. Say Q is the original rationals
and Q_R is that subfield of R. Then 1/2_R is the
element of Q_R which happens to equal the set
of all r in Q which are less than 1/2_Q.

So the symbol "1/2" is being used for two different things.
But what actually happens is this: Once we've constructed
our complete ordered field and found that subfield Q_R
isomoprhic to Q, we _forget_ about Q; from now on
1/2 means 1/2_R.

And that makes the notation confusing when we look
at the definition of 1/2_R as a certain subset of Q.
But that doesn't matter, because we never talk about
that definition! Once we've constructed R as a
complete ordered field the construction disappears
into the background − from that point on everything
we say about R just follows from the fact that it's
a complete ordered field.

> I appreciate that I can define reals using Cauchy sequences, but I am trying to understand the Dedekind cuts.

The definition of the reals using Cauchy sequences has the very same
ambiguity. A real number is by definition a certain equivalence class
of Cauchy sequences of rationals. Now 1/2 could mean one of the
original rationals, or it could mean one of those equivalence classes;
exactly the same problem, with exactly the same resolution.

David C. Ullrich

"Understanding Godel isn't about following his formal proof.
That would make a mockery of everything Godel was up to."
(John Jones, "My talk about Godel to the post−grads."
in sci.logic.)
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