Re: Analysis problem and continuity

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- *Date*: Mon, 08 Sep 2008 12:16:21 −0700

In article <y99xk.9449$cn7.4708@xxxxxxxxxxxxxxxxxxxx>,
"Jon Slaughter" <Jon_Slaughter@xxxxxxxxxxx> wrote:

> "Carl R." <solrac140@xxxxxxxxxxx> wrote in message
> news:fd66f253−0435−4d81−92b8−9a03e85b7a4e@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
>
> > Let f: R − > C be a map from the set of real numbers to the comple
> > numbers.
> > Define phi(x,delta):= sup { | f(s) − f(t) | : s, t in (x−delta, x
> > +delta)}
> > Let r(x):= inf { phi(x,delta) : delta >0 }
> >
> > Prove that
> >
> > a) f is continuous in x iff r(x) = 0.
> >
> > So here's my work.
> >
> > Asumme f is continuous in x then for all e>0 we can find delta >0 such
> > that
> > if |y−x|< delta then |f(y) − f(x) | < epsilon.
> > But if |y−x|< delta then y,s are in (x−delta,x+delta) , thus
> > phi(x,delta): = sup { epsilon } = epsilon.
> > Thus r(x) := inf { epsilon : epsilon > 0} = 0 because epsilon>0.
>
> This is the general idea but the proof is pretty weak and not formulated in
> the "general style".
>
> Let phi(x, d) = sup{|f(s) − f(t)|} for s and t in (x − d, x + d)

phi has already been defined; no need to include it in the proof.
Also, "for s and t ..." should be included in the { }.

> suppose f is continuous,
>
> |s − t| < d ==> |f(s) − f(t)| < e

No. Let e > 0 first. Then there exists d ...

> then 0 <= phi(x,d) < e (this is actually a bit vague... it is true exactly because sup(f(x)) takes on its maximum... there is a theorem and it happens because f(x) is continuous)

Actually it's false. A continuous function on a closed bounded interval assumes its maximum. But (x − d, x + d) is open. The most you can say here is phi(x,d) <= e. Which of course is good enough.

> hence 0 <= r(x) < phi(x,d) < e
>
> since e can be chosen arbitrarily we have r(x) = 0.
>
> This is a bit better, but not necessarily the best. It's logic is much clear than what you have said. Your's is a bit more "intuitive" but it also shows and it is not the standard method of proving things.
>
> i.e,., your last 2 statements are not common. It is almost exactly what I said but you skip the steps:
>
>> phi(x,delta): = sup { epsilon } = epsilon.
>
> why does phi(x,delta) = sup{epsilon}? Surely it can only be <= epsilon
>
>> Thus r(x) := inf { epsilon : epsilon > 0} = 0 because epsilon>0.
>
> the same here... why does r(x) = inf{epsilon} = 0?
>
> You must always go back to the definitions!!!!!!!!
>
> inf(X) <= X <= sup(X)!!!

(it is because epsilon can be chosen arbitrarily small, which you can see how I wrote it.

> Where I'm stuck is the part that if r(x) = 0 then f is continuous in x. Can you please help?

always start with your definitions:

inf{phi(x,d)} = 0

implies that phi(x,d) can be made arbitrarily small.

e.g., there exists a delta that makes it smaller than we wish

i.e.,

phi(x,d) < e for some d

but now phi(x,d) = sup{|f(s) − f(t)|}

but this is easy

|f(s) − f(t)| <= phi(x,d) < e

so we can choose |s − t| < d

which will make r(x) < e

(definition of infimum)

which is just

|f(s) − f(t)| <= phi(x,d) < e

which shows that f(x) is continuous.

I'll leave you to put it all together.

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