D is integral domain and D[t] is PID, how to show D is a field

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D is an integral domain (commutative ring with no zero divisors).

D[t] is Principal Ideal Domain (all ideals are generated by single elements of D[t]).

Then how can one show that D is a field?

Just some suggestion on "how" to show would be helpful. Like method of attack.

My feeling is I should start by picking random nonzero element from D[t] then show that if it is not a unit we get a contradiction. So I should be able to get an ideal that is generated by more than one element.

Side question: Are all ideals of rings generated by elements? The book describes ideals in general, then describes principal ideals, then proves that they are ideals. Then it gives an example of an ideal that is not a principal ideal, which happens to be an ideal generated by two elements. So I presume there are cases of rings with ideals generated by 3 or more elements. Now, must all ideals be generated by some number of elements of the ring?

And is there a common name for ideals generated by 2 elements (something like secondary ideal)?

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