Re: The First Variation of a PDF

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- *From*: William Elliot <marsh@xxxxxxxxxxxxxxxxxx>
- *Date*: Tue, 14 Oct 2008 04:36:13 −0700

On Mon, 13 Oct 2008, junoexpress wrote:

> On Oct 13, 2:50 am, William Elliot <ma...@xxxxxxxxxxxxxxxxxx> wrote:
>
> > On Sun, 12 Oct 2008, junoexpress wrote:
> >
> > > Consider a pdf having compact support on [−X,+X] for some given
> > > positive real X, that is a function of a real parameter c. When c=0,
> > > the pdf is symmetric about x=0 and the pdf takes a very simple form.
> > >
> > > When c is non−zero, the form of the pdf is non−symmetric about x=0
> > > becomes quite complicated.
> > >
> > > I want to understand the effect c has when it is small but non−zero,
> > > on the bias of x.
> > >
> > > One way I considered was to construct an approximation to the pdf:
> >
> > f(x,c) =~ f(x,0) + c.f_c(x,0).

When c > 0, then for f_c(x,0) to be negative, assuming a
smooth x curve >= 0, then f(x,c) > 0 so you've got lee way
to assume the approximation is not negative.

Similar in reverse with 0 < c.

> > Now the RHS of (1) integrates to 1, but it is probably not a
> > pdf
> > (since it is not obvious the first order approx is
> > non−negative).

> integral(−oo,oo) f(x,0) dx + integral(−oo,oo) c.f_c(x,c) dx
> = 1 + c.integral(−oo,oo) f_c(x,0) dx = 1

> My problem is that although the function integrates to one, the
> function is not guaranteed to be a pdf because it has no guarantee of
> being non−negative at all points in its domain.

Have you found examples (see my thoughts above) where it is negative?
Yes if |c| is large enought. However with small c, the approximaation
I'd think should remain non−negative.

> So now, I'm trying to compute things like a bias with a function that is
> not a pdf, and so how do I know what I am really getting is the bias? So
> there's a point of confusion (on my part) as to what one is *gaining* by
> making an approximation of this sort and OTOH, what one is *losing* by
> not using a true pdf.

I don't know. Now were I to alter a metric to see what effect
it has and can only approximate the alteration, wouldn't I want
the approximation to be a metric? If it wasn't, then how could
I judge the effect it had on the space? It would be hard.

> That probably is the best I can do at explaining my concern. Maybe I'm
> nitpicking here, but this bothers me, and I have to think that this is a
> standard problem that people who know math well must have come across.

If for all y, g(y) >= 0, then is there some c > 0
with g(0) + c.g'(c) >= 0?

That's the first simple step.
Now for each x, let g(y) = f(x,y).

Since the support for f(x,y) as a function of x is compact,
can we find a c > 0 with for all x in support
f(x,0) + c.f_y(x,c) >= 0. Outside the support,

isn't f(x,y) for each y, flat, ie zero? Thus
can we assume f_y(x,y) = 0 when x not in support?

Does this have the assumption that for each y, the
pdf of f(x,y) has the same compact support?

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