

Re: Wave decompositions

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On Sat, 18 Oct 2008 06:00:14 EDT, riderofgiraffes
<mathforum.org_am@xxxxxxxxxxxxxxxxxxx> wrote:

I'm trying to figure out whether a wave ...
some physical basis ... has some essential
"canonical" decomposition into sinusoid
waves

(snip)

Any periodic function (with various generous
assumptions that are covered by your "physical basis"
comment) is uniquely decomposable into a linear
combination of sin and cos functions by using Fourier.

This says that any "real, physical wave" really
is (in some sense) made up of sine waves (of some
amplitude and phase).

Wrong! or at least not quite right.

Some time ago:

On Tue, 06 Feb 2007 01:47:55 GMT, andy everett <vze2q...@xxxxxxxxxxxxx>
wrote:

<http://groups.google.com/group/sci.math/msg/6f38ae8ab0baed5a?dmode=source>

If a saw tooth wave can be represented by an alternating sum of sin
waves can a sin wave be represented by some sum of saw tooth waves?

Lee Rudolph wrote:

Now that I know the answer is yes it is easy to sketch the alternating
sum of the first couple of "triangle" waves with wavelength L , $L/3$, $L/5$,
... such a sum starts to "get smooth"

Re: Wave decompositions

Thanks for an amusing challenge. You seem to have found your answer but it took me a while to re-orient myself to waveform harmonics to see the answer. Its a lot like division. The first order sawtooth has sub-harmonics of period $1/2$, $1/3$, $1/4$ etc. Use a second sawtooth of $1/2$ amplitude to suppress the second harmonic. Keeping track of the new harmonics added to the result of this subtraction, just keep subtracting the next remaining subharmonic until your sum starts to "get smooth"

Netting this out--any waveform can be decomposed into any number of orthogonal components. As to whether there is one unique decomposition--that is one excellent question.

There is the Wold Decomposition Theorem. This theorem states that any real-world process can be decomposed into a deterministic component and a noise process. "The noise process can be modeled as the output of a linear filter, excited at its input by a white noise signal."

That is the closest relevant wisdom I am aware of. If seriously pursued, I would start by finding a proof of Wold's theorem and see if it can be adapted to the purpose.